Section 2.1 Inductive and Deductive Reasoning

Inductive Reasoning is the process of drawing a general conclusion by observing a pattern of specific instances. This conclusion is called a hypothesis or conjecture.

1. Show how inductive reasoning can be used to determine the digit in the ones' place (last digit) of this number without actually computing the entire answer.

Inductive Procedure

i) List the pattern

ii) Determine the cycle size

iii) take the problems exponent ÷ cycle size

iv) use the remainder of the division problem to determine the ones digit

a. 4^{97}

b. 3^{24}

c. 7^{15}

Inductive reasoning is limited by the fact that no amount of examples that something is true will prove that it is ALWAYS true. We suspect it is true but we really haven't proved it. If at some point someone finds just 1 **counterexample**, then our conjecture (hypothesis) is proved to be false.

Ex. List a counter example for the following statement: "All prime numbers are odd." _______

Statements, Connectives, Quantifiers   Section 2.2

Statement: in logic a statement is a declarative sentence that is either true or false. We represent statements by lower case letters such as ***p, q***, or ***r***.

Examples of things that are **NOT statements**: Questions, commands, exclamations, and paradoxes are not statements because they do not have the quality of being "true" or "false".

Paradox: a sentence that cannot be assigned a truth value because it contradicts itself. Example: "This statement is false."

Truth value: The quality of being "true" or "false" is the statement's truth value.

Which of the following are **statements** in the mathematical sense?

	Statement (in mathematics)	Truth Value
This year February has 29 days.		
$3 \times 2 = 4 + 2$		
$\{1, 2, 3\} = \{4, 5, 6\}$		
Do you have a car?		
All rules have exceptions		

Types of Statements:

- **Simple Statement:** a statement that contains a single idea. Example: It is a sunny day.
- **Compound Statement**: a compound statement contains several ideas combined together. The words used to join the ideas of a compound statement together are called **connectives**.

5 Logical Connectives:

- **And** ("but" is sometimes used in place of "and")
  as in "I am going, but Mary is not."
- **Or**
- **Not**
- **If , then** Regardless of order in everyday language, "IF" introduces the hypothesis and "THEN" introduces the conclusion. In symbols, the hypothesis "IF", ALWAYS goes first.
  Example: If it rains, I will not go. I will not go if it rains. Both are written the same way in symbols.
- **If and Only If**

Memorize these:

Logical Connectives and their Special Mathematical Names and Symbols

word or phrase	Special Math Name	Symbol
and (both, the overlap)	conjunction	$\wedge$ (like intersection in sets)
or (one or the other, or both)	disjunction,	$\vee$ (like union in sets)
if . . . then (implies)	conditional	$\rightarrow$
If and only if (iff)	biconditional	$\leftrightarrow$
not	negation	~

In symbolic logic, we let letters stand for statements, the way we let letters stand for numbers in algebra.

Translating the compound sentence: Today is Friday and I have a test.

Let P = Today is Friday
Q = I have a test.

In symbols:

Quiz Yourself #4-5 (p. 86, 87 of text) Write each statement in symbolic form:

***d*: I will buy a DVD player.**
***i*: I will buy an iPod.**

(a) I will not buy a DVD player or I will not buy an iPod. ___________________

(b) I will not buy a DVD player and I will buy an iPod. ___________________

***f*: I fly to Houston**
***q*: I will qualify for frequent flyer miles**

(a) If I do not fly to Houston, then I will not qualify for frequent flyer miles.

____

(b) I fly to Houston if and only if I will qualify for frequent flyer miles.

____

Negation applies only to the thing it is immediately next to.

$\sim P \wedge Q$ means ____

$\sim(P \wedge Q)$ means ____

Clues for parentheses:

- **"It is false that" or "It is not true that" (everything after the word 'that' is in parentheses)**
- **Commas (either the phrase before the comma or the phrase after the comma will be in parentheses – which ever one is compound.)**
- **"Neither A nor B" means "not" (A or B)**

Important Counter-Intuitive Fact: (don't let this trip you up)

$\sim(A \wedge B) = \sim A \vee \sim B$ (and is not equal to $\sim A \wedge \sim B$ like you might think).

Write each statement in symbolic form given that
P = Today is Monday
Q = Tomorrow is Wednesday
R = Tomorrow is Tuesday
T = Today is Tuesday

a. It is false that today is Monday or tomorrow is Wednesday.

b. Neither is today Monday nor is tomorrow Wednesday.

c. If today is Monday, then tomorrow is Tuesday or tomorrow is Wednesday.

d. Today is Tuesday if and only if tomorrow is Wednesday.

e. Tomorrow is Wednesday implies that today is Monday. (we can write false statements)

Assignment Due Friday 2/5:

Complete #31-34 on p. 81

Read Section 2.2 pp. 83-90

Complete #2, 5, 6, 11, 14, 17, 21, 23, 25, 30, 31, 35, 37, 39, 43, 45, 54, 55, 67 on pp. 90-92

Finish Guided Notes pp. 25-28