

## 110 Final Exam Review

Sample problems from unit 1 (ch 6 algebra), unit 2 (ch 2 sets), and unit 3 (ch 12 counting and probability).  
New material from sections 6.5 and 6.8 has not been included, nor material from unit 4 (ch 13 statistics).

1. Simplify:

a.  $\frac{20}{3} \div \frac{5}{6}$

$$\frac{20}{3} \cdot \frac{6}{5} = 8$$

b.  $3^2 + 5(-2) - (-4)$

$$9 - 10 + 4$$

$$-1 + 4 = 3$$

c.  $3 - 4[2x - 3(x - 2)]$

$$3 - 4[2x - 3x + 6]$$

$$3 - 4[-x + 6]$$

$$3 + 4x - 24 = 4x - 21$$

d.  $(\frac{3}{4}x^3 - x^2 + \frac{3}{8}) - (\frac{1}{4}x^2 - 2x + \frac{1}{8})$

$$\frac{3}{4}x^3 - x^2 - \frac{1}{4}x^2 + 2x + \frac{3}{8} - \frac{1}{8}$$

$$\frac{3}{4}x^3 - \frac{5}{4}x^2 + 2x + \frac{1}{4}$$

2. Solve  $P = 2l + 2w$  for  $w$ .

$$\frac{-2l - 2l}{-2l - 2l}$$

$$\frac{P - 2l}{2} = \frac{2w}{2}$$

$$w = \frac{P - 2l}{2} \text{ or } w = \frac{P}{2} - l$$

3. Solve:

a.  $3x + \frac{3}{4} = \frac{1}{2} + x$

$$-x - \frac{3}{4} = \frac{1}{2} - x$$

$$\frac{1}{2} 2x = -\frac{1}{4} (\frac{1}{2})$$

$$x = -\frac{1}{8}$$

b.  $\frac{x + 24 + 35 + 40}{4} = 37$

$$4 \left( \frac{x + 99}{4} \right) = (37) \cdot 4$$

$$x + 99 = 148$$

$$-99 \quad -99$$

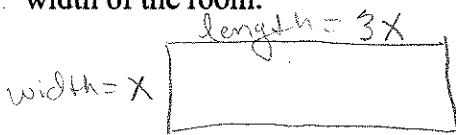
$$x = 49$$

c. Solve for  $h$  if  
 $A = \frac{h(b_1 + b_2)}{2}$

$$2 \cdot A = \frac{h(b_1 + b_2)}{2} \cdot 2$$

$$\frac{2A}{b_1 + b_2} = \frac{h(b_1 + b_2)}{b_1 + b_2}$$

4. The length of a room is 3 times the width. The perimeter is 80 feet. Find the length and width of the room.



$$P = 2x + 2(3x)$$

$$80 = 2x + 6x$$

$$\frac{80}{8} = \frac{8x}{8} \Rightarrow x = 10$$

width = 10 ft  
length = 30 ft

6. Solve.  $7 - \frac{3}{4}x < 13$

$$-7 \quad -7$$

$$\left(-\frac{4}{3}\right) \cdot \frac{3}{4}x < \left(6\right) \cdot \left(-\frac{4}{3}\right)$$

$$x > -18$$

5. If a customer is charged \$190.19 for an item where the sales tax rate is 4.5%, what was the original price? Let  $x$  = original price

$$x + 0.045x = 190.19$$

$$\frac{1.045x}{1.045} = \frac{190.19}{1.045}$$

$$x = 182$$

7. A syllabus gives the following weights for the final grade: midterm: 20%, final exam: 50%, quizzes: 30%. Toby received a score of 70 on the midterm, but only has an average of 20 on quizzes. What does Toby need to achieve on the final to get at least a C in the course?

$$0.2(70) + 0.5(x) + 0.3(20) \geq 70$$

$$14 + 0.5x + 6 \geq 70$$

$$20 + 0.5x \geq 70$$

$$-20 \quad -20$$

$$\frac{0.5x}{0.5} \geq \frac{50}{0.5} \Rightarrow x \geq 100$$

8. Find the slope of the line through  $(-2, 7)$  and  $(3, -3)$ .

$$m = \frac{7 - (-3)}{-2 - 3} = \frac{10}{-5} = -2$$

10. The table at right gives several points on the graph of a line. Which of the following points will also be on the graph of the line?

- a.  $(3, -7)$  b.  $(-3, 9)$  c.  $(4, -3)$  d.  $(-4, 21)$

9. Find the slope, x-intercept, and y-intercept of  $2x - 5y = 10$ .  
 $\rightarrow -5y = -2x + 10 \rightarrow y = \frac{2}{5}x - 2$   
 x-intercept:  $(5, 0)$  y-intercept:  $(0, -2)$   
 slope:  $\frac{2}{5}$

$$\frac{2x}{2} - 5(0) = \frac{10}{2}$$

$$x = 5$$

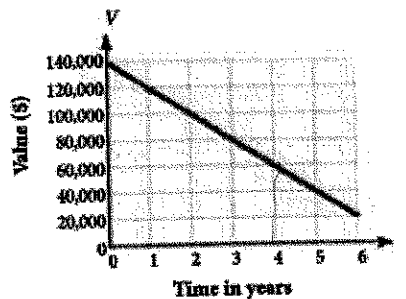
$$2(0) - 5y = 10$$

$$-5y = \frac{10}{-5}$$

$$y = -2$$

x	y
-2	13
-1	9
0	5
1	1
2	-3

11. Elliot buys a new forklift for his business. It will cost \$140,000 and will decrease in value each year. The graph shows the value of the forklift after the first 6 years of ownership.



- a. How much is the forklift worth after 1 year?

$\$120,000$

- b. After how many years is the forklift worth \$60,000?

4 yrs

- c. Find the equation of this line where  $V$  is the value after  $t$  years.

$$V = -20,000t + 140,000$$

- d. Find the slope. Interpret in this context.

$-20,000$  is the amount of depreciation in  $\$/yr$

- e. Find the y-intercept. Interpret in this context.

$\$140,000$  is the original cost of the forklift

12. With a flat fee of \$50 plus the hourly cost to rent a garden tractor, the 6 hour cost was \$200. If the cost to rent the garden tractor is modeled by an equation of the form  $y = mx + b$ , where  $y$  equals the total cost and  $x$  equals the number of hours,  $m$  represents

- a. \$200  
 Total cost  
 b. fixed cost  
 \$50  
 c. rate of change in cost  
 \$25  
 d. total time rented  
 6

13. Let  $f(x) = 3x + 5$  and  $g(x) = x^2 - 4x$ . Evaluate the following:

a.  $f(-4) = 3(-4) + 5 = -12 + 5 = -7$

b.  $g(-3) = (-3)^2 - 4(-3) = 9 + 12 = 21$

14. Which of the following functions represent the input  $x$  and output  $f(x)$  from the tables of values at right? Write the letter of the matching function for each table.

- a.  $f(x) = x^2 - 5x + 4$   
 b.  $f(x) = 4 - x^2$   
 c.  $f(x) = (x - 4)^2$   
 d.  $f(x) = 4^x$   
 e.  $f(x) = 4^{1-x}$   
 f.  $f(x) = 4^{x-1}$

b	a	d	e
x	y	x	y
-2	0	1	0
0	4	4	1
2	0	0	4

$16 - 20 + 4$

1. The following table shows the earned degrees in the U.S. in the 2005-2006 academic year, classified by level and by the gender of the degree recipient. A person is chosen at random.

	Bachelor's	Master's	Professional	Doctorate	Total
Female	855	356	44	27	1282
Male	631	238	44	29	942
	1486	594	88	56	2224

- Find the probability that the degree recipient received a bachelor's degree.
- Find the probability that the degree recipient is male.
- Find the probability that the person is male and received a bachelor's degree.
- Find the probability that the person is male or received a bachelor's degree.
- If the degree recipient is female, find the probability that she received a professional degree.
- If the person received a master's degree, find the probability that the degree recipient is female.
- Find the probability that a person received a professional or doctorate degree.

$P(\text{Bach}) = \frac{1486}{2224} = \frac{743}{1112}$   
 $P(\text{Male}) = \frac{942}{2224} = \frac{471}{1112}$   
 $P(\text{M} \cap \text{B}) = \frac{631}{2224}$   
 $P(\text{M} \cup \text{B}) = \frac{942 + 1486 - 631}{2224} = \frac{1797}{2224}$   
 $P(\text{P} | \text{F}) = \frac{44}{1282} = \frac{22}{641}$   
 $P(\text{F} | \text{M}) = \frac{356}{594} = \frac{178}{297}$

$P(\text{P} \cup \text{D}) = \frac{88 + 56}{2224} = \frac{144}{2224} = \frac{9}{139}$

2.a. Six pieces of paper are numbered 1,2,3,4,5,6. Three papers are drawn at random, without replacement.

- Find the probability that a 1 is drawn first, a 4 second, and a 6 third.

$\frac{1}{6} \cdot \frac{1}{5} \cdot \frac{1}{4} = \frac{1}{120}$

- Find the probability that two even numbers and one odd number is drawn.

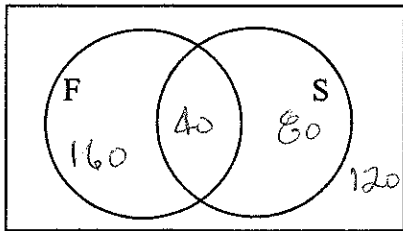
$\frac{3}{6} \cdot \frac{2}{5} \cdot \frac{3}{4} \cdot 3$   
*(this can happen 3 ways)*  
 $\frac{54}{120} = \frac{9}{20}$

- How many outcomes would have a sum greater than 13?

$6+5+3, 5+6+3, 6+3+5, 5+3+6, 3+5+6, 5+6+4, 6+5+4, 4+5+6, 3+6+5, 5+4+6, 6+4+5, 4+6+5$   
 12 arrangements but only 2 odd sums

3. A consumer research survey asked about preferences regarding drinking plain bottle water or a sports drink. The sample included 200 men and 200 women. 280 reported they preferred plain bottled water. Of the group preferring a sports drink, 80 were male and 40 were female.

- Organize the information using a Venn diagram. Let F represent the person was female and S represent the person chose sports drinks.



b. Find  $P(\text{F} \cap \text{S}')$

- Find the probability a person in the study preferred plain bottled water.

$\frac{160 + 120}{400} = \frac{7}{10}$

- Find the probability a person in the study preferred a sports drink.

$\frac{120}{400} = \frac{3}{10}$

- If the consumer is a man, what is the probability that he will prefer bottled water?

$\frac{120}{200} = \frac{3}{5}$

- If the consumer preferred bottled water, what is the probability that the consumer is female?

$\frac{160}{280} = \frac{4}{7}$

4. Set up only; do not simplify nor compute. a. Past data show that 2% of the products from a certain supplier are defective. What is the probability that first two products received are defective?

$\frac{2}{100} \cdot \frac{2}{100}$  or  $0.02 \times 0.02$

5. Set up only; do not simplify nor compute. For a student club with twenty members, ten are math majors and five are business majors.

- a. How many ways can five members be chosen to go to a conference?  $\frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
- b. Find the probability that three math majors and two business majors are chosen.  $\frac{10}{20} \cdot \frac{9}{19} \cdot \frac{8}{18} \cdot \frac{5}{17} \cdot \frac{4}{16}$
- c. Find the probability that all five business majors are chosen for the conference.  $\frac{5}{20} \cdot \frac{4}{19} \cdot \frac{3}{18} \cdot \frac{2}{17} \cdot \frac{1}{16}$
- d. Find the probability that at least one business major is chosen.

$$1 - P(\text{no business}) = 1 - \left( \frac{15}{20} \cdot \frac{14}{19} \cdot \frac{13}{18} \cdot \frac{12}{17} \cdot \frac{11}{16} \right)$$

6. A consumer watch group test three brands (A, B, C) of digital cameras. 40% of the cameras were brand A and 30% of the cameras were brand B. The consumer watch group found that 2% of brand A cameras were defective, 4% of brand B were defective, and 9% of brand C were defective.

- a. Find the probability that a camera was brand A and defective.  $(0.4)(0.02) = 0.008$
- b. Find the probability that a camera was brand C and not defective.

$$91\% \quad [100\% - (40\% + 30\%)] [100\% - 9\%]$$

$$[30\%][91\%] \text{ or } \frac{30}{100} \cdot \frac{91}{100} \text{ or } (0.3)(0.91) = 0.273$$

7. Toss two dice. a. Find the probability that the sum is divisible by 3.  $\rightarrow \frac{12}{36} = \frac{1}{3}$
- b. Find the probability that the sum is an odd number or a number divisible by 3.  $\frac{24}{36} = \frac{2}{3}$

8. How many 5 character passwords are possible if the first two characters must be a letter, the next two characters must be digits, and the last character must be chosen from {#, \$, &, \*}

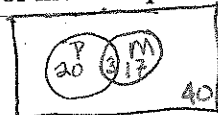
$$26 \cdot 26 \cdot 10 \cdot 10 \cdot 4 = 270,400$$

9. Toss a coin, then toss a single die. Find the Cartesian product to list the sample space as ordered pairs.

$$\{(H_1, H_2, H_3, H_4, H_5, H_6, T_1, T_2, T_3, T_4, T_5, T_6)\}$$

10. In a survey of 80 commuter students, 23 had purchased parking permits and 20 had paid for metered parking. Forty said they had neither purchased a permit nor ever paid for metered parking. Find the probability that a commuter student had purchased a parking permit but still paid for metered parking at some time.

$$P(P \cap M) = \frac{3}{80}$$



11. Six students are giving presentations at a workshop. Joe has requested to be first, since he has another commitment that day. In how many ways can the students be scheduled to give their presentations?

$$1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \text{ ways}$$

1. True or False: Use  $T = \{t, u, r, k, e, y\}$ ,  $P = \{k, e, y\}$ ,  $Q = \{r, u, t\}$

- $t \in T$  True     $P \subseteq T$  True     $\emptyset \subseteq T$  True     $e \in T$  False     $\{e\} \subseteq T$  True

2. Write in roster notation:

a.  $H = \{x | x \in \mathbb{W} \text{ and } x \leq 4\}$

$$\{0, 1, 2, 3, 4\}$$

b.  $\{b : b \in \mathbb{N} \text{ and } b \text{ is a multiple of } 10\}$

$$\{10, 20, 30, 40, \dots\}$$

3. Determine the cardinal number,  $n(A)$ , for each of the following sets.

a.  $A = \{x : x \text{ is a state in the U.S.}\}$

$$n(A) = 50$$

b.  $A = \{1, 0, \emptyset, \{\emptyset\}\}$

$$n(A) = 4$$

4. Given  $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{x: x \text{ is an even whole number less than } 11\}$ ,  
 $B = \{x: 1.5 < x < 4.5\}$ , and  $C = \{7, 8, 9, 10\}$

$7, 8, 9, 10, 2, 3, 4$

- a.  $A \cap B$       b.  $A \cap C$       c.  $A \cup C$       f.  $C' \cup B$       g.  $(C \cup B)'$
- $\{2, 4\}$        $\{8, 10\}$        $\{2, 4, 6, 7, 8, 9, 10\}$        $\{0, 1, 2, 3, 4, 5, 6\}$        $\{0, 1, 5, 6\}$

- d.  $B \cap \emptyset$       h.  $A - B$       i.  $(C - A) \cup A'$       j.  $(A \cap B') \cap C$
- $\emptyset$        $\{0, 6, 8, 10\}$        $\{7, 9\} \cup \{0, 1, 3, 5, 7, 9\}$        $(6, 8, 10) \cap (7, 8, 9, 10) = \{8, 10\}$
- e.  $\emptyset \cup B$        $\{2, 3, 4\}$

5. A student club has eight members.

	Class	Major
Abe	Junior	Music
Ben	Sophomore	Psychology
Cal	Sophomore	International Studies
Dot	Junior	Psychology
Eve	Sophomore	Music
Fred	Junior	International Studies
Gabi	Junior	Psychology
Hank	Junior	Music

Let  $J = \{x: x \text{ is a junior}\}$ ,  $S = \{x: x \text{ is a sophomore}\}$ ,  $M = \{x: x \text{ is majoring in music}\}$ ,  
and  $P = \{x: x \text{ is majoring in psychology}\}$ . Use these defined sets to answer the following:

- a. True or false:  $\{Ben, Cal\} \subseteq P$   
b. True or false:  $\{Ben, Cal\} \subseteq S$   
c. True or false:  $Ben \in (S \cup M)$   
d. True or false:  $Ben \in (M \cap S)$

e. Write the set of students that are juniors but not music majors using set notation and then in roster notation.

$$J \cap M' = \{Dot, Fred, Gabi\}$$

f. Write the set of students that are sophomores or music majors in set-builder notation and then in roster notation.

$$\{x \mid x \in \text{sophomore or music major}\} = \{Eve, Ben, Cal, Abe, Hank\}$$

g. Write the set  $\{Dot, Gabi\}$  in set-builder notation.

$$\{x \mid x \in \text{Junior} \ \& \ x \in \text{Psychology major}\}$$

h. Find  $S \times P$

- $\{(Ben, Ben), (Ben, Dot), (Ben, Gabi), (Cal, Ben), (Cal, Dot), (Cal, Gabi), (Eve, Ben), (Eve, Dot), (Eve, Gabi)\}$

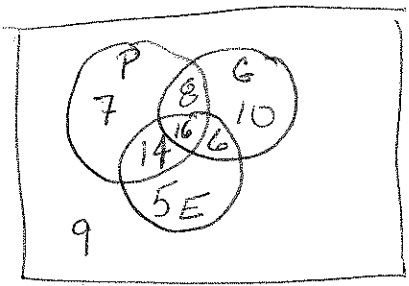
i. List all the subsets of  $M$ .  $2^3 = 8$  subsets

- $\{ \}, \{Abe\}, \{Eve\}, \{Hank\}, \{Abe, Eve\}, \{Abe, Hank\}, \{Eve, Hank\}, \{Abe, Eve, Hank\}$

6. In a group of 75 students, 16 students are taking psychology, geology, and English; 24 students are taking psychology and geology; 30 students are taking psychology and English; 6 students are taking geology and English but not psychology. 7 students are taking only psychology, 40 students are taking geology, and 9 students are not taking any of the three courses.

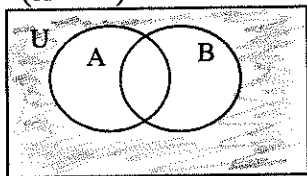
How many students

- a. are taking English? 41  
b. are taking English or psychology, but not geology? 26  
c. are not taking psychology? 30

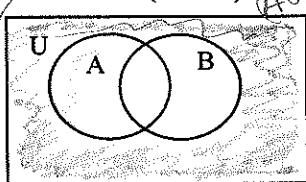


7. In each Venn Diagram below, shade the region associated with the given set.

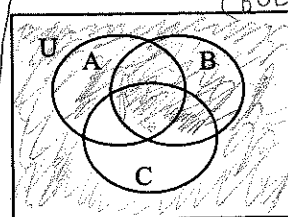
a.  $(A \cup B)' - A$



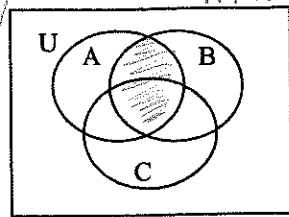
b.  $A \cup (B \cup A)'$



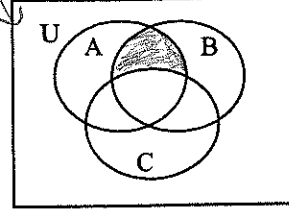
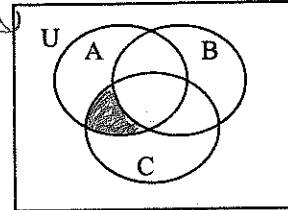
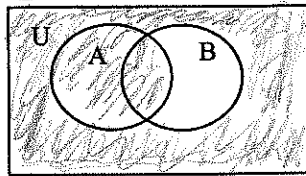
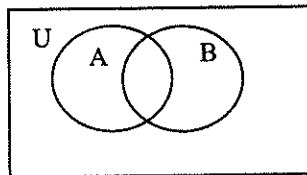
c.  $A \cap (B \cup C)'$



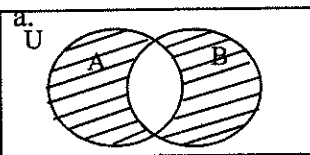
e.  $(A \cap B) - C$



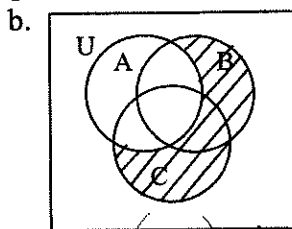
(extras, if needed)



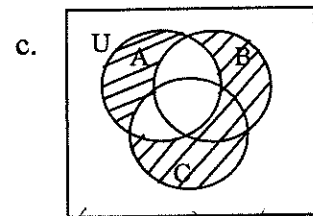
8. Describe the shaded region using set operation notation:



$(A \cup B) - (A \cap B)$



$(B \cup C) - A$  or  $(B \cup C) \cap A'$



$(A \cup B \cup C) - (A \cap B) - (A \cap C)$

9. A certain store has 100 bikes for sale. 35 of these bikes are red, 28 are ten-speed, and 70 cost \$100 or more. There are 14 red bikes that cost less than \$100 and are not ten-speed, but we will find 13 red bikes that cost \$100 or more and are not ten-speed. Twenty of the ten-speed bikes that cost \$100 or more are not red. There are 30 bikes costing \$100 or more that are neither red nor ten-speed.

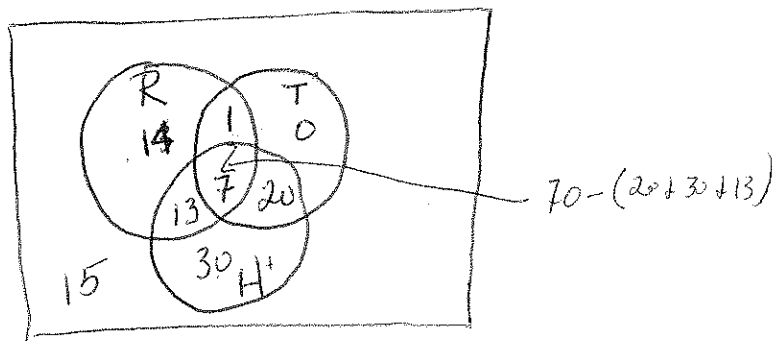
How many bikes are

a. red or ten-speed? 85

b. not red? 65

c. red, but not ten-speed? 27

d. neither red, nor ten-speed? 45

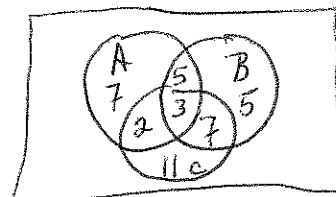


10. Find the number of elements in sets A, B, and C using the given information.

$n(A \cap B) = 8, n(A \cap C) = 5, n(A \cap B \cap C) = 3, n(A - B) = 9,$

$n(B - C) = 10, n(A \cup B) = 29, n(B \cup C) = 33$

$n(A) = 17$   
 $n(B) = 20$   
 $n(C) = 23$



11. Use a one-to-one correspondence to show that  $\{1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \dots\}$  has cardinal number  $\aleph_0$ .

