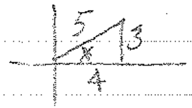
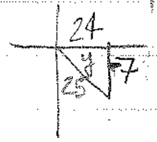
Review Ex p. 711

28. $\sin x = \frac{3}{5}$




$36.9°$

$\cos y = \frac{24}{25}$




$-16.3°$

$\underline{\sin(x+y)} = \sin x \cos y + \cos x \sin y$

$= \frac{3}{5} \cdot \frac{24}{25} + \frac{4}{5} \cdot \frac{-7}{25}$

$= \frac{72}{125} - \frac{28}{125} = \frac{44}{125}$

$\underline{\cos(x-y)} = \cos x \cos y + \sin x \sin y$

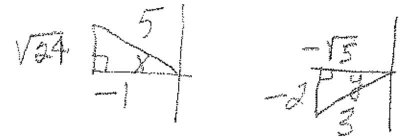
$\frac{4}{5} \cdot \frac{24}{25} + \frac{3}{5} \cdot \frac{-7}{25}$

$\frac{96}{125} - \frac{21}{125} = \frac{75}{125} = \frac{3}{5}$

$\underline{\tan(x+y)} = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{3}{4} + \frac{-7}{24}}{1 - \frac{3}{4} \cdot \frac{-7}{24}}$

$= \frac{\frac{18}{24} - \frac{7}{24}}{\frac{96}{96} + \frac{21}{96}} = \frac{\frac{11}{\cancel{24}} \left(\frac{\cancel{96}}{117}\right)}{\frac{117}{96} \cdot \left(\frac{96}{117}\right)} = \frac{44}{117}$

$x+y$ is in the $\underline{\text{I quadrant}}$




30. $\sin y = \frac{-2}{3}$, $\cos x = -\frac{1}{5}$

x+y is in quadrant IV

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \frac{\sqrt{24}}{5} \cdot \frac{-\sqrt{5}}{3} + \frac{-1}{5} \cdot \frac{-2}{3}$

$\frac{-\sqrt{120}}{15} + \frac{2}{15} = \frac{-2\sqrt{30}+2}{15}$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$-\frac{1}{5} \cdot \frac{-\sqrt{5}}{3} + \frac{\sqrt{24}}{5} \cdot \frac{-2}{3}$

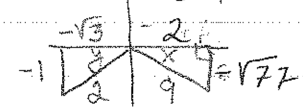
$\frac{\sqrt{5}}{15} - \frac{4\sqrt{6}}{15} = \frac{\sqrt{5}-4\sqrt{6}}{15}$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y} = \frac{-\frac{\sqrt{24}}{1} + \frac{2}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)}{1 - \left(\frac{-\sqrt{24}}{1}\right) \cdot \left(\frac{2}{\sqrt{5}}\right)}$

$= \frac{\frac{-5\sqrt{24}+2\sqrt{5}}{5}}{\left(\frac{5}{5}\right) 1 + \frac{2 \cdot 2\sqrt{6}}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)} = \frac{\frac{-5\sqrt{24}+2\sqrt{5}}{5}}{\frac{5+4\sqrt{30}}{5}} \left(\frac{\frac{5}{5+4\sqrt{30}}}{\frac{5}{5+4\sqrt{30}}}\right) = \frac{-5\sqrt{24}+2\sqrt{5}}{5+4\sqrt{30}} \cdot \frac{5-4\sqrt{30}}{5-4\sqrt{30}}$

$= \frac{-25\sqrt{24}+20\sqrt{720}+10\sqrt{5}-8\sqrt{150}}{25-16(30)} = \frac{-50\sqrt{6}+240\sqrt{5}+10\sqrt{5}-40\sqrt{6}}{-455} = \frac{-90\sqrt{6}+250\sqrt{5}}{-5 \cdot 91} = \frac{18\sqrt{6}-25\sqrt{5}}{91}$

32. $\cos x = \frac{2}{9}$, $\sin y = -\frac{1}{2}$




$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\frac{2}{9} \cdot \frac{-\sqrt{3}}{2} + \frac{-\sqrt{77}}{9} \cdot \frac{-1}{2} = \frac{-2\sqrt{3}-\sqrt{77}}{18}$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$= \left(\frac{-\sqrt{77}}{9}\right)\left(\frac{-\sqrt{3}}{2}\right) + \left(\frac{2}{9}\right)\left(\frac{-1}{2}\right)$

$\frac{\sqrt{231}}{18} - \frac{2}{18} = \frac{\sqrt{231}-2}{18}$

x + y falls in quadrant II

#32) $$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$= \frac{\frac{-\sqrt{77}}{2} + \frac{-1}{-\sqrt{3}}}{1 - \frac{-\sqrt{77}}{2}\left(\frac{1}{\sqrt{3}}\right)} = \frac{-\frac{\sqrt{77}}{2} + \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)}{1 + \frac{\sqrt{77}}{2\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\left(\frac{3}{3}\right)\frac{-\sqrt{77}}{2} + \frac{\sqrt{3}}{3}\left(\frac{2}{2}\right)}{\frac{6}{6} + \frac{\sqrt{231}}{6}} = \frac{\frac{3\sqrt{77}}{6} + \frac{2\sqrt{3}}{6}}{\frac{6+\sqrt{231}}{6}}$$

$$= \frac{\frac{-3\sqrt{77}+2\sqrt{3}}{6}}{\frac{6+\sqrt{231}}{6}}\left(\frac{\frac{6}{6+\sqrt{231}}}{\frac{6}{6+\sqrt{231}}}\right) = \frac{3\sqrt{77}+2\sqrt{3}}{6+\sqrt{231}}$$

$$= \frac{-3\sqrt{77}+2\sqrt{3}}{6+\sqrt{231}}\left(\frac{6-\sqrt{231}}{6-\sqrt{231}}\right)$$

$$= \frac{-18\sqrt{77}+3\sqrt{17,787}+12\sqrt{3}-2\sqrt{693}}{36-231}$$

$$= \frac{-18\sqrt{77}+3\sqrt{77^2}\sqrt{3}+12\sqrt{3}-2\cdot 3\sqrt{77}}{-195}$$

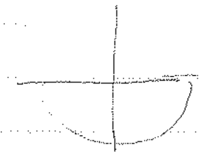
$$= \frac{-18\sqrt{77}+3\overbrace{\sqrt{77}\sqrt{3}}^{231}+12\sqrt{3}-6\sqrt{77}}{-195}$$

$$= \frac{24\sqrt{77}+243\sqrt{3}}{-195}$$

$$= \frac{-\cancel{3}(8\sqrt{77}-81\sqrt{3})}{-\cancel{3}\cdot 65}$$

$$= \frac{8\sqrt{77}-81\sqrt{3}}{65}$$

34. $\cos 2B = \frac{1}{8}, \quad 540^\circ < 2B < 720^\circ$



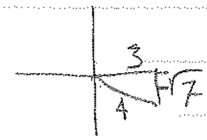
$$\cos 2B = 2\cos^2 B - 1$$

$$\frac{1}{8} = 2\cos^2 B - 1$$

$$+\frac{8}{8} \qquad\qquad +\frac{8}{8}$$

$$\frac{1}{2}\left(\frac{9}{8}\right) = \frac{1}{2}\left(2\cos^2 B\right)$$

$$\sqrt{\frac{9}{16}} = \sqrt{\cos^2 B}$$

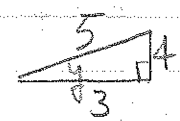



$$\frac{3}{4} = \cos B$$

$$\sin B = \frac{-\sqrt{7}}{4}$$

36. $2y$, given $\sec y = \frac{5}{3}$, $\sin y > 0$

$$\frac{1}{\cos y} = \frac{5}{3}$$




$$\cos y = \frac{3}{5}$$

$$\sin 2y = 2\sin y \cos y$$

$$= \frac{2}{1}\left(\frac{4}{5}\right)\cdot\left(\frac{3}{5}\right) = \frac{24}{25}$$

$$\cos 2y = 2\cos^2 y - 1$$

$$= 2\left(\frac{3}{5}\right)^2 - 1$$

$$= \frac{2}{1}\cdot\frac{9}{25} - \frac{25}{25}$$

$$= \frac{18}{25} - \frac{25}{25} = \frac{-7}{25}$$

Verify

64. $\sin^3\theta = \sin\theta - \cos^2\theta\,\sin\theta$

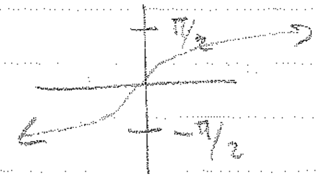
$\sin^3\theta = \sin\theta(1-\cos^2\theta)$

$\sin^3\theta = \sin\theta(\sin^2\theta)$

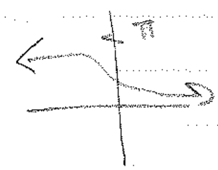
$\sin^3\theta = \sin^3\theta$ ✓verified

T/F A | 68. $y = \tan^{-1}x$

if False exp. | p. 677




p. 678 $y = \cot^{-1}x$




False, the ranges are $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for $\tan^{-1}$ while $(0, \pi)$ for $\cot^{-1}$

#77. Give the exact value of y in radians

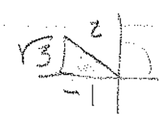
p. 679 $y = \sec^{-1}(-2)$ ; $\cos^{-1}$ $(0, \pi)$

$\sec y = -2$

$\frac{1}{\cos y} = -2$

$\cos y = -\frac{1}{2}$

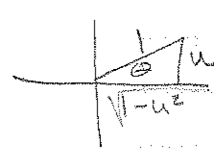
$y = \cos^{-1}\left(-\frac{1}{2}\right)$




$y = 120^\circ = \frac{2\pi}{3}$

100. Write as an alg expression in u, $u > 0$

$\cos\left(\arctan\frac{u}{\sqrt{1-u^2}}\right)$




$\cos(\theta) = \frac{\sqrt{1-u^2}}{1}$

Solve for exact solutions over $[0, 360°)$;
where app. round to nearest tenth

112. $2\tan^2\theta = \tan\theta + 1$

$2\tan^2\theta - \tan\theta - 1 = 0$

Let $x = \tan\theta$

$2x^2 - x - 1 = 0$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)} = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm \sqrt{9}}{4}$$

$x = \frac{1 \pm 3}{4}$ so $x = \frac{1+3}{4} = 1$ or $\frac{1-3}{4} = -\frac{1}{2}$

$\tan\theta = 1$

$\theta = \tan^{-1} 1$

$\theta = 45°$

$+180°$

$225°$

$\tan\theta = -\frac{1}{2}$

$\theta = \tan^{-1}(\frac{1}{2})$

$\theta = -26.6 \Rightarrow 153.4°$

$+180$

$333.4°$

$\{45°, 153.4°, 225°, 333.4°\}$