4/10

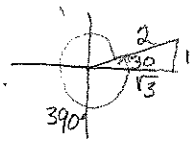
Name Key

Math 143

Quiz 10 – Section 7.4 & 7.5

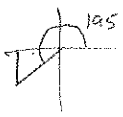
NO CALCULATORS ALLOWED & show your work for full credit.

1. Use a half-angle identity to find the exact value of cos 195° (1 pt)



$\cos 195 = \cos\frac{390}{2} = -\sqrt{\frac{1+\cos 390^\circ}{2}} = -\sqrt{\frac{\left(1+\frac{\sqrt{3}}{2}\right)\frac{1}{2}}{2\left(\frac{1}{2}\right)}} = -\sqrt{\frac{1}{2}+\frac{\sqrt{3}}{4}}$

$= -\sqrt{\frac{2+\sqrt{3}}{4}} = -\frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}} = -\frac{\sqrt{2+\sqrt{3}}}{2}$



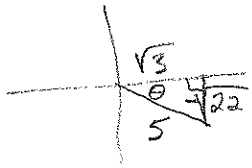
1. $-\frac{\sqrt{2+\sqrt{3}}}{2}$

2. Use identifies to find the value of **sin 2θ**, given $\cos\theta = \frac{\sqrt{3}}{5}$ and $\sin\theta < 0$ (1 pt)

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = 2\left(\frac{-\sqrt{22}}{5}\right)\left(\frac{\sqrt{3}}{5}\right)$

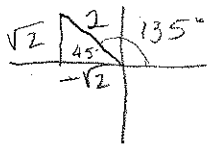
$\sin 2\theta = \frac{-2\sqrt{66}}{25}$



2. $\frac{-2\sqrt{66}}{25}$

3. Give the degree measure of θ if it exists. (1 pt/problem)

a) $\theta = \cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)$



3 a. 135° or $\frac{3\pi}{4}$

b) θ = arcsin 2

$\sin\theta = \sin(\arcsin 2)$

$\sin\theta = 2$ is not in the range.

b. Does not exist

4

4. Choose **two** of the following and **verify the identities.** (3 pts/problem) **Given:**

$$\tan\frac{A}{2}=\pm\sqrt{\frac{1-\cos A}{1+\cos A}},\quad \tan\frac{A}{2}=\frac{\sin A}{1+\cos A},\quad \tan\frac{A}{2}=\frac{1-\cos A}{\sin A},\quad \tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

(3 pts/problem)

i) $\dfrac{\tan^2 x+1}{\tan x\csc^2 x}=\tan x$

$$\frac{\sec^2 x}{\tan x\csc^2 x}=\tan x$$

$$\frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}\cdot\frac{1}{\sin^2 x}}=\tan x$$

$$\frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos x\sin x}}\left(\frac{\cos x\sin x}{1}\right)=\tan x$$

$$\frac{\sin x}{\cos x}=\tan x$$

$$\tan x=\tan x \quad \text{verified}$$

ii) $\cos 2x=\dfrac{2-\sec^2 x}{\sec^2 x}$

$$\cos 2x=\frac{2}{\sec^2 x}-\frac{\sec^2 x}{\sec^2 x}$$

$$\cos 2x=2\cos^2 x-1$$

$$\cos 2x=\cos 2x \quad \text{verified}$$

iii) $\dfrac{\cot^2\theta-1}{1+\cot^2\theta}=1-2\sin^2\theta$ Use $1+\cot^2\theta=\csc^2\theta$

$$\frac{(\csc^2\theta-1)-1}{\csc^2\theta}=1-2\sin^2\theta$$

$$\frac{\csc^2\theta}{\csc\theta}-\frac{2}{\csc^2\theta}=1-2\sin^2\theta$$

$$1-2\sin^2\theta=1-2\sin^2\theta \quad \text{verified}$$