8. $r = \frac{1}{2}$ in., $s = 1$ in.

$s = r\theta \Rightarrow 1 = \frac{1}{2}\theta \Rightarrow \theta = 2$ radians

Review your Unit Circle Measurements to help with problems 9-12

9. $\sin\frac{3\pi}{4}$

Since $\frac{3\pi}{4}$ is in quadrant II, the reference angle is $\pi - \frac{3\pi}{4} = \frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}$. In quadrant II, the sine is positive. Thus, $\sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$. Converting $\frac{3\pi}{4}$ to degrees, we have $\frac{3\pi}{4} = \frac{3}{4}(180°) = 135°$. The reference angle is $180° - 135° = 45°$. Thus, $\sin\frac{3\pi}{4} = \sin 135° = \sin 45° = \frac{\sqrt{2}}{2}$.

10. $\cos\left(-\frac{7\pi}{6}\right)$

$-\frac{7\pi}{6}$ is coterminal with $-\frac{7\pi}{6} + 2\pi = -\frac{7\pi}{6} + \frac{12\pi}{6} = \frac{5\pi}{6}$. Since $\frac{5\pi}{6}$ is in quadrant II, the reference angle is $\pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$. In quadrant II, the cosine is negative. Thus, $\cos\left(-\frac{7\pi}{6}\right) = \cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$.

Converting $\frac{5\pi}{6}$ to degrees, we have $\frac{5\pi}{6} = \frac{5}{6}(180°) = 150°$.

The reference angle is $180° - 150° = 30°$.

Thus, $\cos\left(-\frac{7\pi}{6}\right) = \cos\frac{5\pi}{6} = \cos 150°$

$= -\cos 30° = -\frac{\sqrt{3}}{2}$

11. $\tan\frac{3\pi}{2} = \tan 270°$ is undefined.

12. $\sec\frac{8\pi}{3}$

$\frac{8\pi}{3}$ is coterminal with $\frac{8\pi}{3} - 2\pi = \frac{2\pi}{3}$.

15. $s = \frac{7\pi}{6}$

Since $\frac{7\pi}{6}$ is in quadrant III, the reference angle is $\frac{7\pi}{6} - \pi = \frac{\pi}{6}$. In quadrant III, the sine and cosine are negative.

$\sin\frac{7\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$

$\cos\frac{7\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

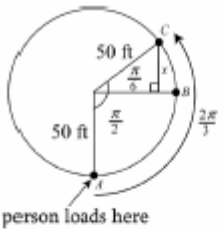
$\tan\frac{7\pi}{6} = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\csc\frac{7\pi}{6} = \frac{1}{\sin\frac{7\pi}{6}} = \frac{1}{-\frac{1}{2}} = -2$

$\sec\frac{7\pi}{6} = \frac{1}{\cos\frac{7\pi}{6}} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\cot\frac{7\pi}{6} = \frac{1}{\tan\frac{7\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

18. (a)



Suppose the person takes a seat at point A. When the person travels $\frac{\pi}{2}$ radians, the person is 50 ft above the ground. When the person travels $\frac{\pi}{6}$ more radians, we can let x be the additional vertical distance traveled:

$\sin\frac{\pi}{6} = \frac{x}{50} \Rightarrow x = 50\sin\frac{\pi}{6} = 50\left(\frac{1}{2}\right) = 25$

Thus, the person traveled an additional 25 ft above the ground, for a total of 75 ft above the ground.

(b) The Ferris wheel goes $\frac{2\pi}{3}$ radians per 30 sec or $\frac{2\pi}{90} = \frac{\pi}{45}$ radian per second.

19. (a) $y = \sec x$ **(b)** $y = \sin x$

(c) $y = \cos x$ **(d)** $y = \tan x$

22. $y = 3 - 6\sin\left(2x + \frac{\pi}{2}\right) = 3 - 6\sin\left[2\left(x + \frac{\pi}{4}\right)\right]$

$= 3 - 6\sin\left[2\left[x - \left(-\frac{\pi}{4}\right)\right]\right]$

(a) The period is $\frac{2\pi}{2} = \pi$.

(b) The amplitude is 6.

(c) The range is $[-3, 9]$.

(d) The y-intercept occurs when $x = 0$.

$$-6\sin\left(2\cdot 0 + \frac{\pi}{2}\right) + 3 = -6\sin\left(0 + \frac{\pi}{2}\right) + 3$$
$$= -6\sin\left(\frac{\pi}{2}\right) + 3$$
$$= -6(1) + 3 = -3$$

(e) The phase shift is $\frac{\pi}{4}$ unit to the left $\left(\text{that is, } -\frac{\pi}{4}\right)$

30. $s(t) = -4\cos 8\pi t$, $a = |-4| = 4$, $\omega = 8\pi$

(a) maximum height = amplitude
$= a = |-4| = 4$ in.

(b) $s(t) = -4\cos 8\pi t = 4 \Rightarrow \cos 8\pi t = -1 \Rightarrow$
$8\pi t = \pi \Rightarrow t = \frac{1}{8}$

The weight first reaches its maximum height after $\frac{1}{8}$ sec.

(c) frequency $= \frac{\omega}{2\pi} = \frac{8\pi}{2\pi} = 4$ cycles per sec;

period $= \frac{2\pi}{\omega} = \frac{2\pi}{8\pi} = \frac{1}{4}$ sec