2. $\sec\theta - \sin\theta\tan\theta = \dfrac{1}{\cos\theta} - \sin\theta\cdot\dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1-\sin^2\theta}{\cos\theta} = \dfrac{\cos^2\theta}{\cos\theta} = \cos\theta$

3. $\tan^2 x - \sec^2 x = \dfrac{\sin^2 x}{\cos^2 x} - \dfrac{1}{\cos^2 x}$

$= \dfrac{\sin^2 x - 1}{\cos^2 x} = -\dfrac{1-\sin^2 x}{\cos^2 x}$

$= -\dfrac{\cos^2 x}{\cos^2 x} = -1$

4. $\cos\dfrac{5\pi}{12} = \cos\left(\dfrac{\pi}{6}+\dfrac{\pi}{4}\right)$

$= \cos\dfrac{\pi}{6}\cos\dfrac{\pi}{4} - \sin\dfrac{\pi}{6}\sin\dfrac{\pi}{4}$

$= \dfrac{\sqrt{3}}{2}\left(\dfrac{\sqrt{2}}{2}\right) - \dfrac{1}{2}\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

5. **(a)** $\cos(270° - x)$

$= \cos 270° \cos x + \sin 270° \sin x$

$= 0\cdot\cos x + (-1)\sin x = 0 - \sin x = -\sin x$

(b) $\tan(\pi + x) = \dfrac{\tan\pi + \tan x}{1 - \tan\pi\tan x} = \tan x$

6. $\sin(-22.5°) = \pm\sqrt{\dfrac{1-\cos(-45°)}{2}} = \pm\sqrt{\dfrac{1-\frac{\sqrt{2}}{2}}{2}}$

$= \pm\sqrt{\dfrac{2-\sqrt{2}}{4}} = \pm\dfrac{\sqrt{2-\sqrt{2}}}{2}$

Since $-22.5°$ is in quadrant IV, $\sin(-22.5°)$ is negative. Thus, $\sin(-22.5°) = -\dfrac{\sqrt{2-\sqrt{2}}}{2}$.

8. Given $\cos\theta = -\dfrac{3}{5}$, $90° < \theta < 180°$

Since θ is in quadrant II, $\sin\theta > 0$, 2θ is in quadrant III or quadrant IV, and

$\dfrac{\pi}{4} < \dfrac{\theta}{2} < \dfrac{\pi}{2} \Rightarrow \dfrac{\theta}{2}$ is in quadrant I. Also

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(-\dfrac{3}{5}\right)^2} = \sqrt{\dfrac{16}{25}} = \dfrac{4}{5}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{4}{5}}{-\frac{3}{5}} = -\dfrac{4}{3}$

(a) $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(-\dfrac{3}{5}\right)^2 - 1 = -\dfrac{7}{25}$

Note that 2θ is in quadrant III because $\cos 2\theta < 0$.

(b) $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\dfrac{4}{5}\right)\left(-\dfrac{3}{5}\right) = -\dfrac{24}{25}$

8.

(c) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2\left(-\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \dfrac{-\frac{8}{3}}{1-\frac{16}{9}}$

$= \dfrac{-24}{9-16} = \dfrac{24}{7}$

(d) $\cos\frac{1}{2}\theta = \sqrt{\dfrac{1+\cos\theta}{2}} = \sqrt{\dfrac{1+\left(-\frac{3}{5}\right)}{2}}$

$= \sqrt{\dfrac{5-3}{10}} = \sqrt{\dfrac{2}{10}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

(e) $\tan\frac{1}{2}\theta = \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{\frac{4}{5}}{1-\frac{3}{5}} = \dfrac{4}{5-3} = 2$

11. Verify $\tan^2 x - \sin^2 x = (\tan x\sin x)^2$ is an identity.

$\tan^2 x - \sin^2 x = \dfrac{\sin^2 x}{\cos^2 x} - \sin^2 x$ 1

$= \dfrac{\sin^2 x - \sin^2 x\cos^2 x}{\cos^2 x}$

$= \dfrac{\sin^2 x\left(1-\cos^2 x\right)}{\cos^2 x} = \dfrac{\sin^2 x\sin^2 x}{\cos^2 x}$

$= \tan^2 x\sin^2 x = (\tan x\sin x)^2$

15. **(a)** $y = \arccos\left(-\dfrac{1}{2}\right) \Rightarrow \cos y = -\dfrac{1}{2}$

Since $0 \le y \le \pi$, $y = \dfrac{2\pi}{3}$.

(b) $y = \sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) \Rightarrow \sin y = -\dfrac{\sqrt{3}}{2}$

Since $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$, $y = -\dfrac{\pi}{3}$.

(c) $y = \tan^{-1} 0 \Rightarrow \tan y = 0$

Since $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$, $y = 0$.

(d) $y = \operatorname{arcsec}(-2) \Rightarrow \sec y = -2$

Since $0 \le y \le \pi$ and $y \ne \dfrac{\pi}{2}$, $y = \dfrac{2\pi}{3}$.

21. $\sin^2\theta = \cos^2\theta + 1$

$\sin^2\theta = 1 - \sin^2\theta + 1$

$2\sin^2\theta = 2 \Rightarrow \sin^2\theta = 1 \Rightarrow \sin\theta = \pm 1$

Over the interval $[0, 360°)$, the equation $\sin\theta = 1$ has one solution, 90°. Over the interval $[0, 2\pi)$, the equation $\sin\theta = -1$ has one solution, 270°. Solution set: $\{90°, 270°\}$

24. $\sqrt{2}\cos 3x - 1 = 0 \Rightarrow \cos 3x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Since $0 \le x < 2\pi, 0 \le 3x < 6\pi$. Thus

$3x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}, \frac{23\pi}{4} \Rightarrow$

$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}$

Solution set: $\left\{\frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}\right\}$