Chapter 8 Test Solutions

1. Find C, given $A = 25.2°$, $a = 6.92$ yd, $b = 4.82$ yd.
Use the law of sines to first find the measure of angle B.

$$\frac{\sin 25.2°}{6.92} = \frac{\sin B}{4.82} \Rightarrow \sin B = \frac{4.82 \sin 25.2°}{6.92} \Rightarrow$$

$$B = \sin^{-1}\left(\frac{4.82 \sin 25.2°}{6.92}\right) \approx 17.3°$$

Use the fact that the angles of a triangle sum to 180° to find the measure of angle C.

$C = 180° - A - B = 180° - 25.2° - 17.3° = 137.5°$

2. Find c, given $C = 118°$, $b = 131$ km, $a = 75.0$ km.
Using the law of cosines to find the length of c.

$$c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow c^2$$
$$= 75.0^2 + 131^2 - 2(75.0)(131)\cos 118°$$
$$\approx 32011.12 \Rightarrow c \approx 178.9 \text{ km}$$

c is approximately 179 km. (rounded to two significant digits)

3. Find B, given $a = 17.3$ ft, $b = 22.6$ ft, $c = 29.8$ ft.
Using the law of cosines, find the measure of angle B.

$$b^2 = a^2 + c^2 - 2ac\cos B \Rightarrow$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{17.3^2 + 29.8^2 - 22.6^2}{2(17.3)(29.8)}$$
$$\approx 0.65617605 \Rightarrow B \approx 49.0°$$

B is approximately 49.0°.

4. $a = 14$, $b = 30$, $c = 40$
We can use Heron's formula to find the area.

$$s = \tfrac{1}{2}(a + b + c) = \tfrac{1}{2}(14 + 30 + 40) = 42$$
$$\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{42(42-14)(42-30)(42-40)}$$
$$= \sqrt{42 \cdot 28 \cdot 12 \cdot 2} = \sqrt{28,224} = 168 \text{ sq units}$$

5. This is SAS, so we can use the formula $\mathcal{A} = \frac{1}{2}zy\sin X$.

$$\mathcal{A} = \frac{1}{2} \cdot 6 \cdot 12 \sin 30° = \frac{1}{2} \cdot 6 \cdot 12 \cdot \frac{1}{2} = 18 \text{ sq units}$$

6. Since $B > 90°$, b must be the longest side of the triangle.

(a) $b > 10$

(b) none

(c) $b \le 10$

21. **(a)** $3(\cos 30° + i\sin 30°) = 3\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$

(b) $4 \text{ cis } 40° = 3.06 + 2.57i$

(c) $3(\cos 90° + i \sin 90°) = 3(0 + 1 \cdot i) = 0 + 3i = 3i$

23. Find all the fourth roots of $-16i = 16(\cos 270° + i\sin 270°)$.

Since $r^4(\cos 4\alpha + i\sin 4\alpha) = 16(\cos 270° + i\sin 270°)$, then we have

$r^4 = 16 \Rightarrow r = 2$ and $4\alpha = 270° + 360° \cdot k \Rightarrow$

$\alpha = \frac{270° + 360° \cdot k}{4} = 67.5° + 90° \cdot k$, k any integer. If $k = 0$, then $\alpha = 67.5°$.

If $k = 1$, then $\alpha = 157.5°$.
If $k = 2$, then $\alpha = 247.5°$.
If $k = 3$, then $\alpha = 337.5°$.

The fourth roots of $-16i$ are

$2(\cos 67.5° + \sin 67.5)$,
$2(\cos 157.5° + i\sin 157.5°)$,
$2(\cos 247.5° + i\sin 247.5°)$, and
$2(\cos 337.5° + i\sin 337.5°)$.

If you are looking for more practice try problem 24 on p. 819

24.

(a) $(0, 5)$

$r = \sqrt{0^2 + 5^2} = \sqrt{0 + 25} = \sqrt{25} = 5$

The point (0, 5) is on the positive y-axis. Thus, $\theta = 90°$. One possibility is (5, 90°). Alternatively, if $\theta = 90° - 360° = -270°$, a second possibility is (5, –270°).

(b) $(-2, -2)$

$r = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

Since θ is in quadrant III,

$\theta = \tan^{-1}\left(\frac{-2}{-2}\right) = \tan^{-1} 1 = 225°$. One possibility is $(2\sqrt{2}, 225°)$. Alternatively, if $\theta = 225° - 360° = -135°$, a second possibility is $(2\sqrt{2}, -135°)$.