Name Key

Math 143
In-Class Exam 3

Show your work for full credit & simplify your solutions. No calculators allowed

1. Write each function value as a cofunction if a complementary angle. (2 pts/problem)

a. $\sec 70^\circ =$ $\csc 20^\circ$

$\csc(90-70^\circ)$

b. $\tan \frac{2\pi}{9} =$ $\cot\left(\frac{\pi}{2}-\frac{2\pi}{9}\right) = \cot\left(\frac{5\pi}{18}\right)$

$\frac{9\pi}{18} - \frac{4\pi}{18}$

2. Find one angle θ that satisfies each of the following. (3 pts/problem)

a. $\sin(\theta + 40^\circ) = \cos(\frac{\theta}{2})$

$\cos(90-(\theta+40)) = \cos\left(\frac{\theta}{2}\right)$

$\cos(50-\theta) = \cos\left(\frac{\theta}{2}\right)$

$50-\theta = \frac{\theta}{2}$

$+\theta \quad +\frac{2}{2}\theta$

$\frac{2}{3}(50) = \frac{3\theta}{2}\left(\frac{2}{3}\right) \Rightarrow \theta = \frac{100}{3} = 33\frac{1}{3}^\circ$

b. $\csc\theta = \sec(3\theta - 26^\circ)$

$\sec(90-\theta) = \sec(3\theta-26)$

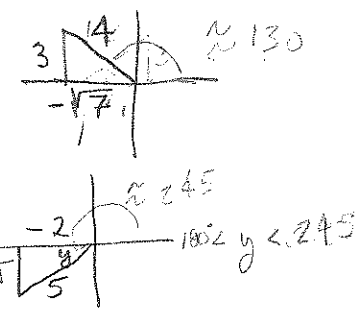
$90-\theta = 3\theta - 26$

$+26+\theta \quad +\theta+26$

$\frac{116}{4} = \frac{4\theta}{4}$

$\theta = 29^\circ$

3. Given $\sin x = \frac{3}{4}$, $\cos y = \frac{-2}{5}$, x is in quadrant II, y is in quadrant III.



a. Find $\cos(x - y)$ (5 pts)

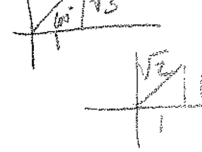
$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\left(\frac{-\sqrt{7}}{4}\right)\left(\frac{-2}{5}\right) + \left(\frac{3}{4}\right)\left(\frac{-\sqrt{21}}{5}\right) = \frac{\sqrt{7}}{10} - \frac{3\sqrt{21}}{20}$ or

$\frac{2\sqrt{7}}{20} + \frac{-3\sqrt{21}}{20} = \frac{2\sqrt{7}-3\sqrt{21}}{20}$

b. What quadrant does $x + y$ fall in? (2 pts) I

4. Find the exact value of each of the following (4 pts/problem)

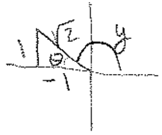


a. $\sin 105^\circ$ (Hint $105^\circ = 60^\circ + 45^\circ$)

$\sin(60+45) = \sin 60 \cos 45 + \cos 60 \sin 45$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$

b. $y = \sec^{-1}(-\sqrt{2})$ (find y in radian measure)



$\sec y = \sec(\sec^{-1}(-\sqrt{2}))$

$\sec y = -\sqrt{2}$

$\frac{1}{\cos y} = -\sqrt{2}$

$\cos y = \frac{1}{-\sqrt{2}}$

$y = \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$y = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

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4. Continued - Find the exact value of the following (4 pts)

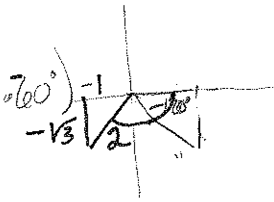
c. $\sin 40^\circ \cos 160^\circ - \cos 40^\circ \sin 160^\circ$

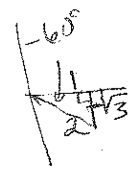
$\sin(40-160) = \sin(-120^\circ) = \sin(2\cdot -60^\circ)$

$= 2\sin(-60)\cos(-60)$

$= 2\left(\frac{-\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$





5. If the $\cos 2\theta = \frac{13}{32}$, $90^\circ < \theta < 180^\circ$ (4 pts/problem)

a. Find the $\sin \theta$.

$\cos 2\theta = 1 - 2\sin^2\theta$

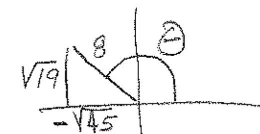
$\frac{13}{32} = 1 - 2\sin^2\theta$

$-\frac{32}{32} \quad -\frac{32}{32}$

$\left(-\frac{1}{2}\right) -\frac{19}{32} = \left(-\frac{1}{2}\right) - 2\sin^2\theta$

$\sqrt{\frac{19}{64}} = \sqrt{\sin^2\theta} \Rightarrow \sin\theta = \frac{\sqrt{19}}{8}$

b. Find the $\cos \theta$.



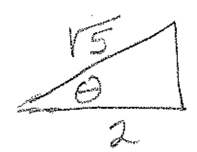
$\cos\theta = \frac{-\sqrt{45}}{8} = \frac{-3\sqrt{5}}{8}$

6. Evaluate each of the following, if it exists. (2 pts/problem)

a. $\arcsin(\sin \frac{\pi}{3})$

$\frac{\pi}{3}$

b. $\cos(\operatorname{arccot} 2)$



$\cos(\theta) = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$\cos(\operatorname{arccot}\theta) = \frac{2\sqrt{5}}{5}$

c. $\arctan(-1)$

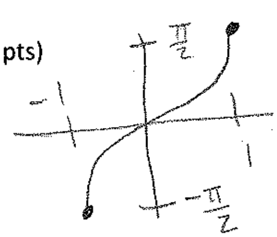
-45 or $-\frac{\pi}{4}$ or $\frac{3\pi}{4}$

d. $\cos^{-1}(3)$

Does not exist

7. Consider the inverse sin function, $y = \sin^{-1} x$.

a. Sketch the function. (3 pts)



b. What ~~is~~ are coordinates of the function at its minimum? (2 pts) $\left(-1, -\frac{\pi}{2}\right)$

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8. Determine whether each statement is true or false. If false, correct the statement to make it true.

(2 pts/problem)

a. $\sin(-\theta) = \sin\theta$

False $\quad \sin(-\theta) = -\sin(\theta)$

b. $\sec x \cos x$ simplifies to $\cos^2 x$

False $\quad \sec x(\cos x) = \frac{1}{\cos x} \cdot \frac{\cos x}{1} = 1$

c. If $\sin\theta\cos\theta - \sin\theta = 0$ then the solution set is $180°n$, where n is any integer.

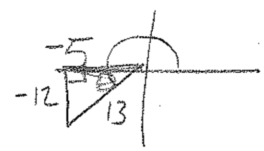
$\sin\theta(\cos\theta - 1) = 0$, $\quad \sin\theta = 0$, $\quad \cos\theta - 1 = 0$

$\theta = 0 + 180n \quad \cos\theta = 1 \rightarrow \theta = \cos^{-1}(1) = 0$

True

9. Given $\tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$ and $\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$

Find $\tan\frac{\theta}{2}$ when $\sin\theta = \frac{-12}{13}$ and $180° < \theta < 270°$ (5 pts)



$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{\frac{-12}{13}}{1+\frac{-5}{13}} = \frac{\frac{-12}{13}}{\frac{13}{13}-\frac{5}{13}} = \frac{\frac{-12}{13}}{\frac{8}{13}} \cdot \frac{\frac{13}{8}}{\frac{13}{8}} = -\frac{12}{8} = \frac{-3}{2}$$

$$\tan\frac{\theta}{2} = \frac{-3}{2}$$

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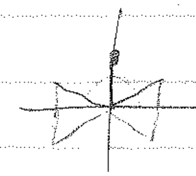
Solve $\underline{two}$ of the following

$4\cos^4 x - 1$

#10 a. $(2\cos^2 - 1)(2\cos^2 x + 1) = 0$

$\cos 2x = 0$, $2\cos^2 x + 1 = 0$ No solution

$\cos^{-1}(\cos 2x) = \cos^{-1} 0$



$\frac{1}{2}(2x) = \frac{\pi}{2}\left(\frac{1}{2}\right)$

$$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

b. $2\cos^2 x + \sin x - 2 = 0$

$2(1 - \sin^2 x) + \sin x - 2 = 0$

$\cancel{2} - 2\sin^2 x + \sin x - \cancel{2} = 0$

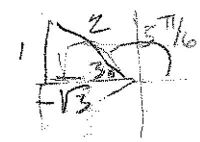
$\sin x(-2\sin x + 1) = 0$

$\sin x = 0$, $-2\sin x + 1 = 0 \Rightarrow \frac{-2\sin x}{-2} = \frac{-1}{-2}$

$ -1 \; -1$

$\{0, \pi/6, \frac{5\pi}{6}\}$ $\qquad \sin^{-1}(\sin x) \overset{\sin^{-1}}{=} \left(\frac{1}{2}\right)$

$x =$

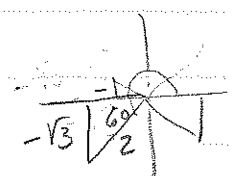


c. $2\sin x + \sqrt{3} = 0$

$ -\sqrt{3} \; -\sqrt{3}$

$\frac{2\sin x}{2} = \frac{-\sqrt{3}}{2}$

$\sin^{-1} \sin x \overset{\sin^{-1}}{=} -\frac{\sqrt{3}}{2}$



$$\boxed{x = \frac{4\pi}{3}, \frac{5\pi}{3}}$$

11. Verify one of the following:

a) $\frac{1}{\sin t - 1} + \frac{1}{\sin t + 1} = -2 \tan t \sec t$

$\frac{\sin t + 1 + \sin t - 1}{\sin^2 t - 1} = -2 \tan t \sec t$

$\frac{2 \sin t}{-\cos^2 t} = -2 \tan t \sec t$

$\frac{2 \sin t}{-\cos t} \cdot \frac{1}{\cos t} = -2 \tan t \sec t$

$-2 \tan t \cdot \sec t = -2 \tan \sec t$

b. $1 - \frac{\sin^2 x}{1 + \cos x} = \cos x$

$\frac{1 + \cos x}{1 + \cos x} - \frac{1 - \cos^2 x}{1 + \cos x} = \cos x$

$\frac{\cos x + \cos^2 x}{1 + \cos x} = \cos x$

$\frac{\cos (\cancel{1 + \cos x})}{\cancel{1 + \cos x}} = \cos x$