Section 8.6 Day 2 Solutions

32. $x^3+1=0 \Rightarrow x^3=-1$

We have $r=1$ and $\theta=180°$.

$x^3=-1=-1+0i=1(\cos 180° + i\sin 180°)$

Since $r^3(\cos 3\alpha + i\sin 3\alpha)$

$=1(\cos 180° + i\sin 180°)$, then we have

$r^3=1 \Rightarrow r=1$ and $3\alpha = 180° + 360° \cdot k \Rightarrow$

$\alpha = \dfrac{180° + 360° \cdot k}{3} = 60° + 120° \cdot k$, k any integer. If $k=0$, then $\alpha = 60° + 0° = 60°$.

If $k=1$, then $\alpha = 60° + 120° = 180°$.

If $k=2$, then $\alpha = 60° + 240° = 300°$.

Solution set:

$\{\cos 60° + i\sin 60°, \cos 180° + i\sin 180°,$

$\cos 300° + i\sin 300°\}$ or

$\left\{\frac{1}{2}+\frac{\sqrt{3}}{2}i, -1, \frac{1}{2}-\frac{\sqrt{3}}{2}i\right\}$

35. $x^3-8=0 \Rightarrow x^3=8$

We have $r=8$ and $\theta=0°$.

$x^3=8=8+0i=8(\cos 0° + i\sin 0°)$

Since $r^3(\cos 3\alpha + i\sin 3\alpha)$

$=8(\cos 0° + i\sin 0°)$, then we have

$r^3=8 \Rightarrow r=2$ and $3\alpha = 0° + 360° \cdot k \Rightarrow$

$\alpha = \dfrac{0° + 360° \cdot k}{3} = 0° + 120° \cdot k = 120° \cdot k$, k any integer. If $k=0$, then $\alpha = 0°$.

If $k=1$, then $\alpha = 120°$. If $k=2$, then $\alpha = 240°$.

Solution set:

$\{2(\cos 0° + i\sin 0°), 2(\cos 120° + i\sin 120°),$

$2(\cos 240° + i\sin 240°)\}$ or

$\{2, -1+\sqrt{3}i, -1-\sqrt{3}i\}$

37. $x^4 + 1 = 0 \Rightarrow x^4 = -1$

We have $r = 1$ and $\theta = 180°$.

$x^4 = -1 = -1 + 0i = 1(\cos 180° + i \sin 180°)$

Since $r^4(\cos 4\alpha + i \sin 4\alpha)$

$= 1(\cos 180° + i \sin 180°)$, then we have

$r^4 = 1 \Rightarrow r = 1$ and $4\alpha = 180° + 360° \cdot k \Rightarrow$

$\alpha = \dfrac{180° + 360° \cdot k}{4} = 45° + 90° \cdot k$, k any integer. If $k = 0$, then $\alpha = 45° + 0° = 45°$.

If $k = 1$, then $\alpha = 45° + 90° = 135°$.

If $k = 2$, then $\alpha = 45° + 180° = 225°$.

If $k = 3$, then $\alpha = 45° + 270° = 315°$.

Solution set:

$\{\cos 45° + i \sin 45°, \cos 135° + i \sin 135°, \cos 225° + i \sin 225°, \cos 315° + i \sin 315°\}$ or

$\left\{\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i, -\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i, -\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i, \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i\right\}$

38. $x^4 + 16 = 0 \Rightarrow x^4 = -16$

We have $r = 16$ and $\theta = 180°$.

$x^4 = -16 = -16 + 0i = 16(\cos 180° + i \sin 180°)$

Since $r^4(\cos 4\alpha + i \sin 4\alpha)$

$= 16(\cos 180° + i \sin 180°)$, then we have

$r^4 = 16 \Rightarrow r = 2$ and $4\alpha = 180° + 360° \cdot k \Rightarrow$

$\alpha = \dfrac{180° + 360° \cdot k}{4} = 45° + 90° \cdot k$, k any integer. If $k = 0$, then $\alpha = 45° + 0° = 45°$.

If $k = 1$, then $\alpha = 45° + 90° = 135°$.

If $k = 2$, then $\alpha = 45° + 180° = 225°$.

If $k = 3$, then $\alpha = 45° + 270° = 315°$.

Solution set:

$\{2(\cos 45° + i \sin 45°), 2(\cos 135° + i \sin 135°), 2(\cos 225° + i \sin 225°), 2(\cos 315° + i \sin 315°)\}$ or

$\{\sqrt{2} + i\sqrt{2}, -\sqrt{2} + i\sqrt{2}, -\sqrt{2} - i\sqrt{2}, \sqrt{2} - i\sqrt{2}\}$

44. $x^3 + 27 = 0 \Rightarrow (x+3)\left(x^2 - 3x + 9\right) = 0$

Setting each factor equal to zero, we have

$x + 3 = 0 \Rightarrow x = -3$ and

$x^2 - 3x + 9 = 0 \Rightarrow$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1} = \frac{3 \pm \sqrt{-27}}{2}$$

$$= \frac{3 \pm 3i\sqrt{3}}{2} = \frac{3}{2} \pm \frac{3\sqrt{3}}{2} i$$

Thus, $x = -3, \frac{3}{2} + \frac{3\sqrt{3}}{2} i, \frac{3}{2} - \frac{3\sqrt{3}}{2} i$. We see that the solutions are the same as Exercise 36.

57. $x^5 + 2 + 3i = 0 \Rightarrow x^5 = -2 - 3i$

$$r = \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13} \Rightarrow$$

$$r^{1/n} = r^{1/5} = \left(\sqrt{13}\right)^{1/5} = 13^{1/10} \approx 1.2924$$

and $\tan\theta = \frac{-3}{-2} = 1.5$

Since θ is in quadrant III, $\theta \approx 236.310°$ and

$$\alpha = \frac{236.310° + 360° \cdot k}{5} = 47.262° + 72° \cdot k,$$

where k is an integer.

$x \approx 1.29239(\cos 47.262° + i \sin 47.262°),$

$1.29239(\cos 119.262° + i \sin 119.2622°),$

$1.29239(\cos 191.262° + i \sin 191.2622°),$

$1.29239(\cos 263.262° + i \sin 263.2622°),$

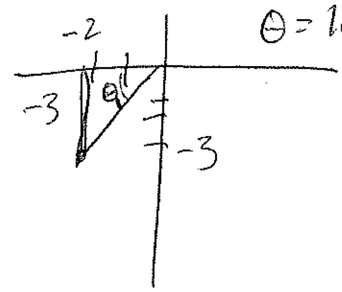
$1.29239(\cos 335.262° + i \sin 335.2622°)$

Solution set:

$\{0.8771 + 0.9492i, -0.6317 + 1.1275i, -1.2675 - 0.2524i, -0.1516 - 1.2835i, 1.1738 - 0.54083i\}$

$\tan\theta_1 = \frac{-3}{-2}$

$\theta_1 = \tan^{-1}(1.5) = 56.31$



$\theta = 180° + 56.31°$

$\theta = 236.31°$