$\overline{34}$

Name Key

Math 143

Take Home Exam 3 – Due Nov. 17th

Show your work for full credit & simple your solutions. Additional paper with work can be attached.

1. Solve the equation over the interval $[0^\circ, 360^\circ)$ rounding your solutions to the nearest tenth of a degree. (4 pts)

$$5\cot^2\theta - \cot\theta = 2 \rightarrow 5\cot^2\theta - \cot\theta - 2 = 0$$

Let $X = \cot\theta$

$$5X^2 - X - 2 = 0$$

$$X = \frac{+1 \pm \sqrt{(-1)^2 - 4(5)(-2)}}{2(5)} = \frac{1 \pm \sqrt{1+40}}{10}$$

$$X = \frac{1+\sqrt{41}}{10} \approx \boxed{0.7403},\quad X = \frac{1-\sqrt{41}}{10} \approx -0.5403$$

$\cot\theta = 0.7403 \rightarrow \frac{1}{\tan\theta} = 0.7403$

$\tan\theta = \frac{1}{0.7403}$

$\theta = \tan^{-1}\left(\frac{1}{0.7403}\right)$

$\boxed{\theta = 53.5^\circ}$

$\boxed{\theta = 233.4^\circ}$

$\cot\theta = -0.5403$

$\frac{1}{\tan\theta} = -0.5403$

$\theta = \tan^{-1}\left(-\frac{1}{0.5403}\right)$

$\theta = -61.6^\circ$

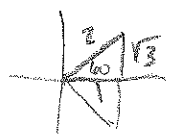
$+180$

$\boxed{\theta = 118.4^\circ}$

$\boxed{\theta = 298.4}$

2. Find **all** the solutions to $\cos\theta = \frac{1}{2}$. (4 pts)

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$



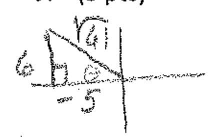
$\boxed{\{60^\circ + 360n,\ 300 + 360n \text{ where } n \text{ is any integer}\}}$

3. Use an identity to write $\sqrt{\dfrac{1+\cos 58^\circ}{2}}$ as a single trigonometric function. (2 pts)

$\cos\frac{58}{2} = \underline{\cos 29^\circ}$

4. Find the exact value of $\sin\theta$ if $\cot\theta = \dfrac{-5}{6}$ and $\cos\theta < 0$. (3 pts)

$$\sin\theta = \frac{6}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}} = \frac{6\sqrt{61}}{61}$$



5. Give the exact value of $\tan(\arccos \frac{\sqrt{3}}{2} + \arcsin \frac{-3}{5})$ (4 pts)

See next page for step-by-step work

$$\frac{25\sqrt{3} - 48}{39}$$

6. The equation $\theta_2 = \tan^{-1}(x\, C/G)$ gives the phase angle (in degrees) of impedance in the parallel portion of a distributed constant circuit. Find θ_2 if $x = 190$ radians per second, $C = 0.04\ \mu F$ per kilometer, and $G = 1.97\ \mu$siemens per kilometer. (2 pts)

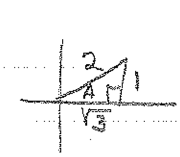
See next page for work

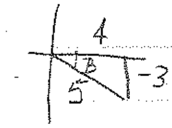
$$\theta = 75.47^\circ$$

Take Home Exam 3

#5. $\tan\left(\arccos \frac{\sqrt{3}}{2} + \arcsin -\frac{3}{5}\right)$

Let $A = \arccos \frac{\sqrt{3}}{2}$ and $B = \arcsin -\frac{3}{5}$





$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$= \frac{\frac{1}{\sqrt{3}}\left(\frac{4}{4}\right) + -\frac{3}{4}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)}{1 - \frac{1}{\sqrt{3}} \cdot \left(-\frac{3}{4}\right)}$$

$$= \frac{\frac{4}{4\sqrt{3}} - \frac{3\sqrt{3}}{4\sqrt{3}}}{\frac{4\sqrt{3}}{4\sqrt{3}} + \frac{3}{4\sqrt{3}}}$$

$$= \frac{\left(\frac{4-3\sqrt{3}}{\cancel{4\sqrt{3}}}\right)\left(\frac{\cancel{4\sqrt{3}}}{4\sqrt{3}+3}\right)}{\frac{\cancel{4\sqrt{3}+3}}{\cancel{4\sqrt{3}}}\left(\frac{\cancel{4\sqrt{3}}}{\cancel{4\sqrt{3}+3}}\right)}$$

$$= \left(\frac{4-3\sqrt{3}}{4\sqrt{3}+3}\right)\left(\frac{4\sqrt{3}-3}{4\sqrt{3}-3}\right)$$

$$= \frac{16\sqrt{3} - 12 - 12 \cdot 3 + 9\sqrt{3}}{16 \cdot 3 - 9}$$

$$= \frac{25\sqrt{3} - 48}{48 - 9}$$

$$= \boxed{\frac{25\sqrt{3} - 48}{39}}$$

#6. $\theta_2 = \tan^{-1}(x \cdot C/G)$

$\theta_2 = \tan^{-1}(190 \cdot 0.04/1.97)$

$\theta_2 = \tan^{-1}(3.8578)$

$\theta_2 = 75.4682°$ $\quad \theta_2 = 1.3172$

7. a. $1 - \tan^2 x = \dfrac{\cos 2x}{\cos^2 x}$

$1 - \tan^2 x = \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}$

$1 - \tan^2 x = \dfrac{\cos^2 x}{\cos^2 x} - \dfrac{\sin^2 x}{\cos^2 x}$

$1 - \tan^2 x = 1 - \tan^2 x$ Verified

b. $\cot \dfrac{x}{2} - \tan \dfrac{x}{2} = 2 \cot x$

$\dfrac{1}{\frac{\sin x}{1 + \cos x}} - \dfrac{1 - \cos x}{\sin x}$

$\dfrac{1 + \cos x}{\sin x} + \dfrac{-1 + \cos x}{\sin x}$

$\dfrac{2 \cos x}{\sin x} =$

$2 \cot x = 2 \cot x$ Verified

c. $\sin 2x - \cot x = -\cot x \cdot \cos 2x$

$2 \sin x \cos x - \dfrac{\cos x}{\sin x} =$

$\left(\dfrac{\sin x}{\sin x}\right) \dfrac{2 \sin x \cos x}{1} - \dfrac{\cos x}{\sin x} =$

$\dfrac{2 \sin^2 x \cdot \cos x - \cos x}{\sin x} =$

$\dfrac{\cos x \,(2 \sin^2 x - 1)}{\sin x} =$

$\dfrac{\cos x \,(-\cos 2x)}{\sin x} =$

$\dfrac{-\cos x}{\sin x} \cdot \left(\dfrac{\cos 2x}{1}\right) =$

$-\cot x \cdot \cos 2x = -\cot x \cdot \cos 2x$ Verified

d) $\frac{\sin(x+y)}{\cos(x-y)} = \frac{\cot x + \cot y}{1 + \cot x \cdot \cot y}$

$$= \frac{\frac{\cos x}{\sin x} + \frac{\cos y}{\sin y}}{1 + \frac{\cos x}{\sin x} \cdot \frac{\cos y}{\sin y}}$$

$$= \frac{\frac{\sin y \cos x + \cos y \sin x}{\sin x \cdot \sin y}}{\frac{\sin x \cdot \sin y}{\sin x \sin y} + \frac{\cos x \cdot \cos y}{\sin x \sin y}}$$

$$= \frac{\frac{\sin y \cos x + \cos y \sin x}{\cancel{\sin x \cdot \sin y}} \left(\frac{\cancel{\sin x \sin y}}{\sin x \sin y + \cos x \cos y}\right)}{\frac{\cancel{\sin x \sin y + \cos x \cdot \cos y}}{\cancel{\sin x \sin y}} \left(\frac{\cancel{\sin x \sin y}}{\cancel{\sin x \sin y + \cos x \cos y}}\right)}$$

$$= \frac{\sin y \cos x + \cos y \sin x}{\sin x \sin y + \cos x \cos y}$$

$$\frac{\sin(x+y)}{\cos(x-y)} = \frac{\sin(y+x)}{\cos(x-y)}$$ verified