Activity 10: How Many Arrangements?

PURPOSE Introduce permutations and combinations.

MATERIALS Colored squares

GROUPING Work individually or in pairs.

GETTING STARTED One square can be arranged in a row in one way.

Two different color squares can be arranged in a row in two distinct ways.

 and 

1. a. Use three different color squares. How many distinct ways can you arrange these three squares in a row? Record each arrangement as you make it. Enter the total number in the table.

Number of Squares	1	2	3	4
Number of Arrangements	1	2		

   b. How do you know you have found all the possible ways to arrange the squares?

   c. How is the number of arrangements of three squares related to the number of arrangements of two squares?

2. a. How many distinct ways do you think four different color squares can be arranged in a row? Why?

   b. Use four different color squares. Make as many distinct arrangements as you can with the squares. Record each arrangement as you make it.

   c. Enter the total number of arrangements in the table. How does this compare with your prediction in Part a? How is it related to the number of arrangements of three squares?

3. If you had n different color squares, how many distinct arrangements could you make? Justify your answer.

ARRANGEMENTS WITH LIKE SQUARES

1. a. Use two red squares and one green square. How many distinct ways can you arrange these three squares in a row? Record each arrangement as you make it.

   b. If all the squares in part a had been different colors, how many arrangements would have been possible?

   c. The number of arrangements in part a is what fraction of the number in part b? How is this fraction related to the number of red squares?

   d. Explain why the number of arrangements of two red squares and one green square should be this fraction of the number of arrangements of three squares.

2. a. Repeat Exercise 1 parts a–c using three red squares and one green square.

   b. If you started with four red squares and one green square, how many distinct ways do you think the five squares could be arranged in a row? Show how you made your prediction.

   c. Use four red squares and one green square. Make as many distinct arrangements as you can with the squares. How many arrangements did you find? How does this compare with the number you predicted in part b?

3. How many distinct ways can you arrange each of the following sets of squares in a row?

   a. 2 red, 1 green, 1 blue b. 2 red, 2 green c. 3 red, 2 green

4. a. The number of arrangements of each set of squares in Exercise 3 is what fraction of the number of arrangements that would be possible if all of the squares had been different colors?

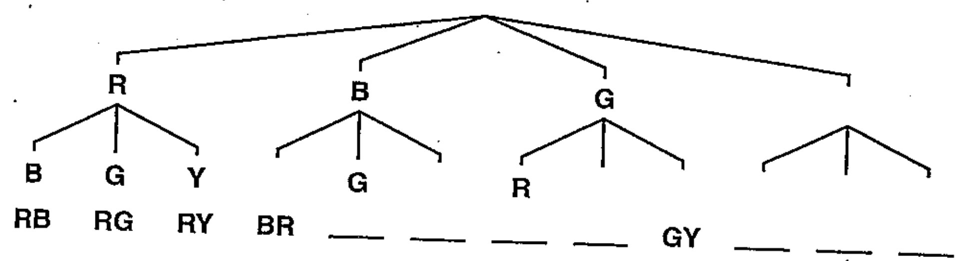
   b. How is each fraction in part a related to the numbers of each color square in the set?

5. Suppose you have a set of squares some of which have the same colors. Explain how to find the number of distinct ways the squares can be arranged in a row.

8

In the preceding exercises, you have been working with permutat
A *permutation* is an arrangement of a group of items in which the order is important.

1. a. Suppose you have one red, one blue, one green, and one yellow square. Complete the tree diagram below to find the number of permutations that can be formed by choosing two of the squares and arranging them in a row.



b. How could you predict the number of permutations in Part a without using a tree diagram or listing them?

c. How many permutations can be formed by choosing three of the squares? Explain how to find the number of permutatio without listing the arrangements.

d. Make a tree diagram to check your answer in part c.

2. Now, suppose you want to find how many different pairs of squares can be chosen from a set containing one red, one blue, one green, and one yellow square.

a. Is the pair consisting of the red square and the blue square (the pair RB) different from the pair BR? Explain.

b. Does changing the order of the squares in a pair change the pair itself?

A selection of items in which order is not important is a *combination*.

3. a. For each arrangement in the tree diagram in Exercise 1 part a on the previous page, cross out all the other arrangements that contain the same pair of squares. How many different combinations remain?

   b. The number of combinations is what fraction of the number of permutations?

   c. How is the fraction related to the number of squares in a pair?

4. Suppose you want to choose a group of three squares from a set containing one red, one blue, one green, and one yellow square.

   a. Use your tree diagram from Exercise 1d. Cross out the duplicate arrangements.

   b. How many different combinations are possible in this situation?

   c. What fraction of the number of permutations is this?

   d. How is the fraction related to the number of squares in a group?

5. How can you find the number of different possible subsets (combinations) of m items that can be selected from a set of n items?