Activity 7: Paper-Scissors-Rock

PURPOSE Introduce the use of a matrix to find experimental probabilities and a tree diagram to calculate theoretical probabilities. Reinforce fair games and use a matrix to find theoretical probabilities.

GROUPING Work in pairs.

GETTING STARTED The *Paper-Scissors-Rock* game has been popular for many years. The two-player game is played as follows:

- Each player makes a fist.
- On the count of three, each player shows either *scissors* by showing two fingers, *paper* by showing four fingers, or *rock* by showing a fist.
- If scissors and paper are shown, the player showing scissors wins, since the scissors cut paper.
- If scissors and rock are shown, the player showing rock wins, since a rock breaks the scissors.
- If paper and rock are shown, the player showing paper wins, since paper wraps a rock.

PROBABILITIES USING A MATRIX

1. Do you think *Paper-Scissors-Rock* is a fair game? Explain.

2. Play the game 45 times. Each player should tally the outcomes in a *matrix* like the one below.

		Your Partner		
		Paper	Scissors	Rock
You	Paper			
	Scissors			
	Rock			

3. Use the data in your matrix to calculate the following experimental probabilities.
P(you win) = ______ P(your partner wins) = ______ P(tie) = ______

4. Use the probabilities in Exercise 3 to decide whether *Paper-Scissors-Rock* is a fair game. Explain your decision.

MORE EXPERIMENTAL PROBABILITIES

1. Use the data in your matrix to calculate the following probabilities.
   a. P(you show rock) = ______
   b. P(your partner shows paper) = ______
   c. P(you show rock) × P(your partner shows paper) = ______
   d. P(you show rock and your partner shows paper) = ______
2. How do your answers to Exercise 1 parts c and d compare?
3. Is P(you show scissors and your partner shows rock) about equal to P(you show scissors) × P(your partner shows rock)? Explain.

THEORETICAL PROBABILITIES USING A MATRIX

You can determine if the game is fair without conducting an experiment.

1. Complete the matrix at the right.
2. If the players choose the sign they show randomly, each of the nine outcomes in the matrix is *equally likely*. Find the following probabilities in this case.
   P(A wins) = ______
   P(B wins) = ______
   P(Tie) = ______
3. Based on the probabilities in Exercise 2, is *Paper-Scissors-Rock* a fair game? Explain.

A = A wins
B = B wins
T = Tie

		Player B		
		Paper	Scissors	Rock
Player A	Paper			A
	Scissors		T	
	Rock			

4. Use the matrix to find the following theoretical probabilities.
   a. P(A shows rock) = ______ b. P(B shows paper) = ______
   c. P(A shows rock) × P(B shows paper) = ______
   d. P(A shows rock ∩ B shows paper) = ______
5. How do your answers to Exercise 4 parts c and d compare?
6. How does the theoretical probability in Exercise 4 part d compare with the experimental probability you found in Exercise 1 part d above?

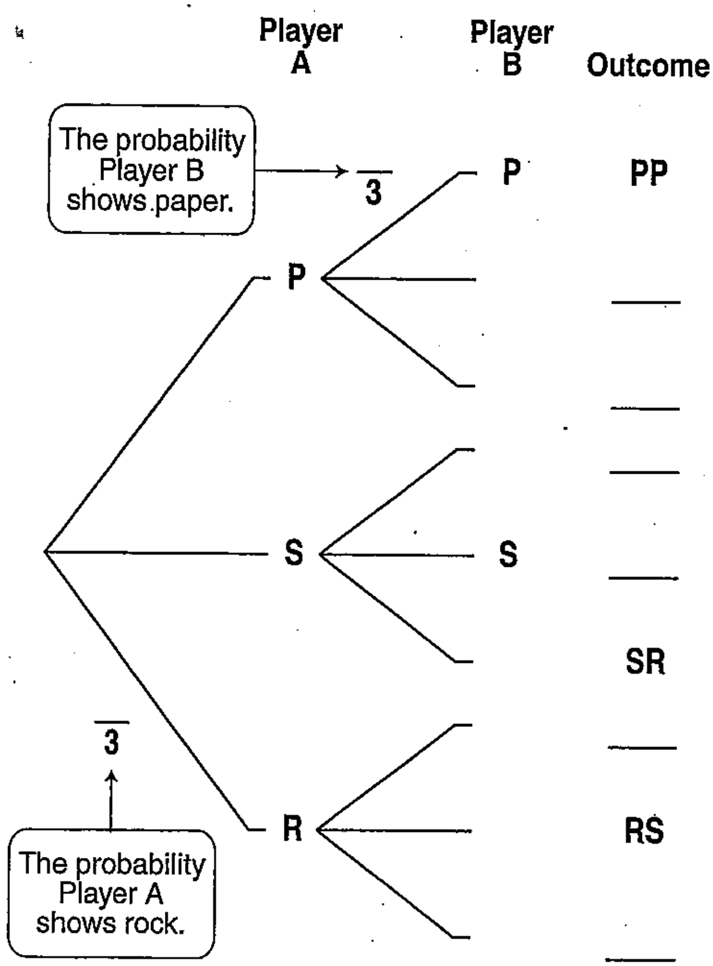
PROBABILITIES USING A TREE DIAGRAM

Since *Paper-Scissors-Rock* can be thought of as a multi-stage experiment, it can be analyzed using a *tree diagram*.

1. Complete the tree diagram at the right.
2. What does the outcome PP in the tree diagram mean?

Consider the path leading to the outcome PP. Since the choices made by Player A and Player B are *independent* of one another, based on the probabilities along the path, we would expect the following:

- In $\frac{1}{3}$ of the games played, Player A will show paper.
- Player B will show paper in $\frac{1}{3}$ of the games in which Player A shows paper $\left(\frac{1}{3} \text{ of } \frac{1}{3}\right)$.



This shows that the probability of each outcome is the product of the probabilities along the path leading to the outcome.

Use the probabilities in the tree diagram to find the following theoretical probabilities.

3. P(PS) = ______
4. P(SP ∪ SR) = ______
5. P(A wins) = ______
6. P(B wins) = ______
7. P(at least one player shows scissors) = ______

Outcome: 2 5 1 5