Minnesota K-12 Academic Standards in Mathematics

April 14, 2007 Revision

Minnesota Department of Education

Sorted by Grade Level

	Strand	Standard	No.	Benchmark
K	Number & Operation	Understand the relationship between quantities and whole numbers up to 31.	0.1.1.1	Recognize that a number can be used to represent how many objects are in a set or to represent the position of an object in a sequence. *For example:* Count students standing in a circle and count the same students after they take their seats. Recognize that this rearrangement does not change the total number. Also recognize that rearrangement typically changes the order in which students are counted.
			0.1.1.2	Read, write, and represent whole numbers from 0 to at least 31. Representations may include numerals, pictures, real objects and picture graphs, spoken words, and manipulatives such as connecting cubes. *For example:* Represent the number of students taking hot lunch with tally marks.
			0.1.1.3	Count, with and without objects, forward and backward to at least 20.
			0.1.1.4	Find a number that is 1 more or 1 less than a given number.
			0.1.1.5	Compare and order whole numbers, with and without objects, from 0 to 20. *For example:* Put the number cards 7, 3, 19 and 12 in numerical order.
		Use objects and pictures to represent situations involving combining and separating.	0.1.2.1	Use objects and draw pictures to find the sums and differences of numbers between 0 and 10.
			0.1.2.2	Compose and decompose numbers up to 10 with objects and pictures. *For example:* A group of 7 objects can be decomposed as 5 and 2 objects, or 3 and 2 and 2, or 6 and 1.
	Algebra	Recognize, create, complete, and extend patterns.	0.2.1.1	Identify, create, complete, and extend simple patterns using shape, color, size, number, sounds and movements. Patterns may be repeating, growing or shrinking such as ABB, ABB, ABB or ●,●●,●●●.
	Geometry & Measurement	Recognize and sort basic two- and three-dimensional shapes; use them to model real-world objects.	0.3.1.1	Recognize basic two- and three-dimensional shapes such as squares, circles, triangles, rectangles, trapezoids, hexagons, cubes, cones, cylinders and spheres.
			0.3.1.2	Sort objects using characteristics such as shape, size, color and thickness.
			0.3.1.3	Use basic shapes and spatial reasoning to model objects in the real-world. *For example:* A cylinder can be used to model a can of soup. *Another example:* Find as many rectangles as you can in your classroom. Record the rectangles you found by making drawings.

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K	Geometry & Measurement	Compare and order objects according to location and measurable attributes.	0.3.2.1	Use words to compare objects according to length, size, weight and position. *For example:* Use same, lighter, longer, above, between and next to. *Another example*: Identify objects that are near your desk and objects that are in front of it. Explain why there may be some objects in both groups.
			0.3.2.2	Order 2 or 3 objects using measurable attributes, such as length and weight.
1	Number & Operation	Count, compare and represent whole numbers up to 120, with an emphasis on groups of tens and ones.	1.1.1.1	Use place value to describe whole numbers between 10 and 100 in terms of groups of tens and ones. *For example:* Recognize the numbers 11 to 19 as one group of ten and a particular number of ones.
			1.1.1.2	Read, write and represent whole numbers up to 120. Representations may include numerals, addition and subtraction, pictures, tally marks, number lines and manipulatives, such as bundles of sticks and base 10 blocks.
			1.1.1.3	Count, with and without objects, forward and backward from any given number up to 120.
			1.1.1.4	Find a number that is 10 more or 10 less than a given number. *For example:* Using a hundred grid, find the number that is 10 more than 27.
			1.1.1.5	Compare and order whole numbers up to 100.
			1.1.1.6	Use words to describe the relative size of numbers. *For example:* Use the words equal to, not equal to, more than, less than, fewer than, is about, and is nearly to describe numbers.
			1.1.1.7	Use counting and comparison skills to create and analyze bar graphs and tally charts. *For example:* Make a bar graph of students' birthday months and count to compare the number in each month.
	Number & Operation	Use a variety of models and strategies to solve addition and subtraction problems in real-world and mathematical contexts.	1.1.2.1	Use words, pictures, objects, length-based models (connecting cubes), numerals and number lines to model and solve addition and subtraction problems in part-part-total, adding to, taking away from and comparing situations.
			1.1.2.2	Compose and decompose numbers up to 12 with an emphasis on making ten. *For example:* Given 3 blocks, 7 more blocks are needed to make 10.
			1.1.2.3	Recognize the relationship between counting and addition and subtraction. Skip count by 2s, 5s, and 10s.
	Algebra	Recognize and create patterns; use rules to describe patterns.	1.2.1.1	Create simple patterns using objects, pictures, numbers and rules. Identify possible rules to complete or extend patterns. Patterns may be repeating, growing or shrinking. Calculators can be used to create and explore patterns. *For example:* Describe rules that can be used to extend the pattern 2, 4, 6, 8, ☐, ☐, ☐ and complete the pattern 33, 43, ☐, 63, ☐, 83 or 20, ☐, ☐, 17.

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1	Algebra	Use number sentences involving addition and subtraction basic facts to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.	1.2.2.1	Represent real-world situations involving addition and subtraction basic facts, using objects and number sentences. *For example:* One way to represent the number of toys that a child has left after giving away 4 of 6 toys is to begin with a stack of 6 connecting cubes and then break off 4 cubes.
			1.2.2.2	Determine if equations involving addition and subtraction are true. *For example*: Determine if the following number sentences are true or false 7 = 7 7 = 8 – 1 5 + 2 = 2 + 5 4 + 1 = 5 + 2.
			1.2.2.3	Use number sense and models of addition and subtraction, such as objects and number lines, to identify the missing number in an equation such as: 2 + 4 = □ 3 + □ = 7 5 = □ – 3.
			1.2.2.4	Use addition or subtraction basic facts to represent a given problem situation using a number sentence. *For example*: 5 + 3 = 8 could be used to represent a situation in which 5 red balloons are combined with 3 blue balloons to make 8 total balloons.
	Geometry & Measurement	Describe characteristics of basic shapes. Use basic shapes to compose and decompose other objects in various contexts.	1.3.1.1	Describe characteristics of two- and three-dimensional objects, such as triangles, squares, rectangles, circles, rectangular prisms, cylinders, cones and spheres. *For example*: Triangles have three sides and cubes have eight vertices (corners).
			1.3.1.2	Compose (combine) and decompose (take apart) two- and three-dimensional figures such as triangles, squares, rectangles, circles, rectangular prisms and cylinders. *For example*: Decompose a regular hexagon into 6 equilateral triangles; build prisms by stacking layers of cubes; model an ice cream cone by composing a cone and half of a sphere. *Another example*: Use a drawing program to find shapes that can be made with a rectangle and a triangle.
		Use basic concepts of measurement in real-world and mathematical situations involving length, time and money.	1.3.2.1	Measure the length of an object in terms of multiple copies of another object. *For example*: Measure a table by placing paper clips end-to-end and counting.

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1	Geometry & Measurement	Use basic concepts of measurement in real-world and mathematical situations involving length, time and money.	1.3.2.2	Tell time to the hour and half-hour.
			1.3.2.3	Identify pennies, nickels and dimes and find the value of a group of these coins, up to one dollar.
2	Number & Operation	Compare and represent whole numbers up to 1000, with an emphasis on place value.	2.1.1.1	Read, write and represent whole numbers up to 1000. Representations may include numerals, addition, subtraction, multiplication, words, pictures, tally marks, number lines and manipulatives, such as bundles of sticks and base 10 blocks.
			2.1.1.2	Use place value to describe whole numbers between 10 and 1000 in terms of groups of hundreds, tens and ones. Know that 100 is ten groups of 10, and 1000 is ten groups of 100. *For example*: Writing 853 is a shorter way of writing 8 hundreds + 5 tens + 3 ones.
			2.1.1.3	Find 10 more or 10 less than any given three-digit number. Find 100 more or 100 less than any given three-digit number. *For example*: Find the number that is 10 less than 382 and the number that is 100 more than 382.
			2.1.1.4	Round numbers up to the nearest 10 and 100 and round numbers down to the nearest 10 and 100. *For example*: If there are 17 students in the class and granola bars come 10 to a box, you need to buy 20 bars (2 boxes) in order to have enough bars for everyone.
			2.1.1.5	Compare and order whole numbers up to 1000.
			2.1.1.6	Use addition and subtraction to create and obtain information from tables, bar graphs and tally charts.
		Demonstrate mastery of addition and subtraction basic facts; add and subtract one- and two-digit numbers in real-world and mathematical problems.	2.1.2.1	Use strategies to generate addition and subtraction facts including making tens, fact families, doubles plus or minus one, counting on, counting back, and the commutative and associative properties. Use the relationship between addition and subtraction to generate basic facts. *For example*: Use the associative property to make ten when adding $5 + 8 = (3 + 2) + 8 = 3 + (2 + 8) = 3 + 10 = 13$.
			2.1.2.2	Demonstrate fluency with basic addition facts and related subtraction facts.
		Demonstrate mastery of addition and	2.1.2.3	Estimate sums and differences up to 100. *For example*: Know that 23 + 48 is about 70.

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2	Number & Operation	subtraction basic facts; add and subtract one- and two-digit numbers in real-world and mathematical problems.	2.1.2.4	Use mental strategies and algorithms based on knowledge of place value to add and subtract two-digit numbers. Strategies may include decomposition, expanded notation, and partial sums and differences. *For example*: Using decomposition, 78 + 42, can be thought of as: 78 + 2 + 20 + 20 = 80 + 20 + 20 = 100 + 20 = 120 and using expanded notation, 34 - 21 can be thought of as: 30 + 4 – 20 – 1 = 30 – 20 + 4 – 1 = 10 + 3 = 13.
			2.1.2.5	Solve real-world and mathematical addition and subtraction problems involving whole numbers with up to 2 digits.
	Algebra	Recognize, create, describe, and use patterns and rules to solve real-world and mathematical problems.	2.2.1.1	Identify, create and describe simple number patterns involving repeated addition or subtraction, skip counting and arrays of objects such as counters or tiles. Use patterns to solve problems in various contexts. *For example*: Skip count by 5 beginning at 3 to create the pattern 3, 8, 13, 18, …. *Another example*: Collecting 7 empty milk cartons each day for 5 days will generate the pattern 7, 14, 21, 28, 35, resulting in a total of 35 milk cartons.
		Use number sentences involving addition, subtraction and unknowns to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.	2.2.2.1	Understand how to interpret number sentences involving addition, subtraction and unknowns represented by letters. Use objects and number lines and create real-world situations to represent number sentences. *For example*: One way to represent $n + 16 = 19$ is by comparing a stack of 16 connecting cubes to a stack of 19 connecting cubes; $24 = a + b$ can be represented by a situation involving a birthday party attended by a total of 24 boys and girls.
			2.2.2.2	Use number sentences involving addition, subtraction, and unknowns to represent given problem situations. Use number sense and properties of addition and subtraction to find values for the unknowns that make the number sentences true. *For example*: How many more players are needed if a soccer team requires 11 players and so far only 6 players have arrived? This situation can be represented by the number sentence $11 - 6 = p$ or by the number sentence $6 + p = 11$.

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2	Geometry & Measurement	Identify, describe and compare basic shapes according to their geometric attributes.	2.3.1.1	Describe, compare, and classify two- and three-dimensional figures according to number and shape of faces, and the number of sides, edges and vertices (corners).
			2.3.1.2	Identify and name basic two- and three-dimensional shapes, such as squares, circles, and triangles, rectangles, trapezoids, hexagons, cubes, rectangular prisms, cones, cylinders and spheres. *For example*: Use a drawing program to show several ways that a rectangle can be decomposed into exactly three triangles.
		Understand length as a measurable attribute; use tools to measure length.	2.3.2.1	Understand the relationship between the size of the unit of measurement and the number of units needed to measure the length of an object. *For example*: It will take more paper clips than whiteboard markers to measure the length of a table.
			2.3.2.2	Demonstrate an understanding of the relationship between length and the numbers on a ruler by using a ruler to measure lengths to the nearest centimeter or inch. *For example*: Draw a line segment that is 3 inches long.
		Use time and money in real-world and mathematical situations.	2.3.3.1	Tell time to the quarter-hour and distinguish between a.m. and p.m.
			2.3.3.2	Identify pennies, nickels, dimes and quarters. Find the value of a group of coins and determine combinations of coins that equal a given amount. *For example*: 50 cents can be made up of 2 quarters, or 4 dimes and 2 nickels, or many other combinations.
3	Number & Operation	Compare and represent whole numbers up to 10,000, with an emphasis on place value.	3.1.1.1	Read, write and represent whole numbers up to 10,000. Representations may include numerals, expressions with operations, words, pictures, number lines, and manipulatives such as bundles of sticks and base 10 blocks.
			3.1.1.2	Use place value to describe whole numbers between 1000 and 10,000 in terms of groups of thousands, hundreds, tens and ones. *For example*: Writing 4,873 is a shorter way of writing the following sums: 4 thousands + 8 hundreds + 7 tens + 3 ones 48 hundreds + 7 tens + 3 ones 487 tens + 3 ones.
			3.1.1.3	Find 1000 more or 1000 less than any given four-digit number. Find 100 more or 100 less than a given four-digit number.

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3	Number & Operation	Compare and represent whole numbers up to 10,000, with an emphasis on place value.	3.1.1.4	Round numbers to the nearest 1000, 100 and 10. Round up and round down to estimate sums and differences. *For example*: 8726 rounded to the nearest 1000 is 9000, rounded to the nearest 100 is 8700, and rounded to the nearest 10 is 8730. *Another example*: 473 – 291 is between 400 – 300 and 500 – 200, or between 100 and 300.
			3.1.1.5	Compare and order whole numbers up to 10,000.
		Add and subtract multi-digit whole numbers; represent multiplication and division in various ways; solve real-world and mathematical problems using arithmetic.	3.1.2.1	Add and subtract multi-digit numbers, using efficient and generalizable procedures based on knowledge of place value, including standard algorithms.
			3.1.2.2	Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Assess the reasonableness of results based on the context. Use various strategies, including the use of a calculator and the relationship between addition and subtraction, to check for accuracy. *For example*: The calculation 117 – 83 = 34 can be checked by adding 83 and 34.
			3.1.2.3	Represent multiplication facts by using a variety of approaches, such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line and skip counting. Represent division facts by using a variety of approaches, such as repeated subtraction, equal sharing and forming equal groups. Recognize the relationship between multiplication and division.
			3.1.2.4	Solve real-world and mathematical problems involving multiplication and division, including both "how many in each group" and "how many groups" division problems. *For example*: You have 27 people and 9 tables. If each table seats the same number of people, how many people will you put at each table? *Another example*: If you have 27 people and tables that will hold 9 people, how many tables will you need?
			3.1.2.5	Use strategies and algorithms based on knowledge of place value and properties of addition and multiplication to multiply a two- or three-digit number by a one-digit number. Strategies may include mental strategies, partial products, the standard algorithm, and the commutative, associative, and distributive properties. *For example*: $9 \times 26 = 9 \times (20 + 6) = 9 \times 20 + 9 \times 6 = 180 + 54 = 234$.

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3	Number & Operation	Understand meanings and uses of fractions in real-world and mathematical situations.	3.1.3.1	Read and write fractions with words and symbols. Recognize that fractions can be used to represent parts of a whole, parts of a set, points on a number line, or distances on a number line. *For example*: Parts of a shape (3/4 of a pie), parts of a set (3 out of 4 people), and measurements (3/4 of an inch).
			3.1.3.2	Understand that the size of a fractional part is relative to the size of the whole. *For example*: One-half of a small pizza is smaller than one-half of a large pizza, but both represent one-half.
			3.1.3.3	Order and compare unit fractions and fractions with like denominators by using models and an understanding of the concept of numerator and denominator.
	Algebra	Use single-operation input-output rules to represent patterns and relationships and to solve real-world and mathematical problems.	3.2.1.1	Create, describe, and apply single-operation input-output rules involving addition, subtraction and multiplication to solve problems in various contexts. *For example*: Describe the relationship between number of chairs and number of legs by the rule that the number of legs is four times the number of chairs.
		Use number sentences involving multiplication and division basic facts and unknowns to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.	3.2.2.1	Understand how to interpret number sentences involving multiplication and division basic facts and unknowns. Create real-world situations to represent number sentences. *For example*: The number sentence $8 \times m = 24$ could be represented by the question "How much did each ticket to a play cost if 8 tickets totaled $24?"
			3.2.2.2	Use multiplication and division basic facts to represent a given problem situation using a number sentence. Use number sense and multiplication and division basic facts to find values for the unknowns that make the number sentences true. *For example*: Find values of the unknowns that make each number sentence true $6 = p \div 9$ $24 = a \times b$ $5 \times 8 = 4 \times t$. *Another example*: How many math teams are competing if there is a total of 45 students with 5 students on each team? This situation can be represented by $5 \times n = 45$ or $\frac{45}{5} = n$ or $\frac{45}{n} = 5$.
	Geometry & Measurement	Use geometric attributes to describe and create shapes in various contexts.	3.3.1.1	Identify parallel and perpendicular lines in various contexts, and use them to describe and create geometric shapes, such as right triangles, rectangles, parallelograms and trapezoids.
			3.3.1.2	Sketch polygons with a given number of sides or vertices (corners), such as pentagons, hexagons and octagons.

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3	Geometry & Measurement	Understand perimeter as a measurable attribute of real-world and mathematical objects. Use various tools to measure perimeter.	3.3.2.1	Use half units when measuring distances. *For example*: Measure a person's height to the nearest half inch.
			3.3.2.2	Find the perimeter of a polygon by adding the lengths of the sides.
			3.3.2.3	Measure distances around objects. *For example*: Measure the distance around a classroom, or measure a person's wrist size.
		Use time, money and temperature to solve real-world and mathematical problems.	3.3.3.1	Tell time to the minute, using digital and analog clocks. Determine elapsed time to the minute. *For example*: Your trip began at 9:50 a.m. and ended at 3:10 p.m. How long were you traveling?
			3.3.3.2	Know relationships among units of time. *For example*: Know the number of minutes in an hour, days in a week and months in a year.
			3.3.3.3	Make change up to one dollar in several different ways, including with as few coins as possible. *For example*: A chocolate bar costs $1.84. You pay for it with $2. Give two possible ways to make change.
			3.3.3.4	Use an analog thermometer to determine temperature to the nearest degree in Fahrenheit and Celsius. *For example*: Read the temperature in a room with a thermometer that has both Fahrenheit and Celsius scales. Use the thermometer to compare Celsius and Fahrenheit readings.
	Data Analysis	Collect, organize, display, and interpret data. Use labels and a variety of scales and units in displays.	3.4.1.1	Collect, display and interpret data using frequency tables, bar graphs, picture graphs and number line plots having a variety of scales. Use appropriate titles, labels and units.
4	Number & Operation	Compare and represent whole numbers up to 100,000, with an emphasis on place value.	4.1.1.1	Read, write and represent whole numbers up to 100,000. Representations include numerals, words and expressions with operations.
			4.1.1.2	Find 10,000 more and 10,000 less than a given five-digit number. Find 1,000 more and 1,000 less than a given five-digit number.
			4.1.1.3	Use an understanding of place value to multiply a number by 10, 100 and 1000.

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4	Number & Operation	Demonstrate mastery of multiplication and division basic facts; multiply multi-digit numbers; solve real-world and mathematical problems using arithmetic.	4.1.2.1	Demonstrate fluency with multiplication and division facts.
			4.1.2.2	Multiply multi-digit numbers, using efficient and generalizable procedures, based on knowledge of place value, including standard algorithms.
			4.1.2.3	Estimate products and quotients of multi-digit whole numbers by using rounding, benchmarks and place value to assess the reasonableness of results in calculations. *For example*: 53 × 38 is between 50 × 30 and 60 × 40, or between 1500 and 2400, and 411/73 is between 400/80 and 500/70, or between 5 and 7.
			4.1.2.4	Solve multi-step real-world and mathematical problems requiring the use of addition, subtraction and multiplication of multi-digit whole numbers. Use various strategies including the relationships between the operations and a calculator to check for accuracy.
			4.1.2.5	Use strategies and algorithms based on knowledge of place value and properties of operations to divide multi-digit whole numbers by one- or two-digit numbers. Strategies may include mental strategies, partial quotients, the commutative, associative, and distributive properties and repeated subtraction. *For example*: A group of 324 students are going to a museum in 6 buses. If each bus has the same number of students, how many students will be on each bus?
		Represent and compare fractions and decimals in real-world and mathematical situations; use place value to understand how decimals represent quantities.	4.1.3.1	Represent equivalent fractions using fraction models such as parts of a set, fraction circles, fraction strips, number lines and other manipulatives. Use the models to determine equivalent fractions.
			4.1.3.2	Locate fractions on a number line. Use models to order and compare whole numbers and fractions, including mixed numbers and improper fractions. *For example*: Locate $\frac{5}{3}$ and $1\frac{3}{4}$ on a number line and give a comparison statement about these two fractions, such as "$\frac{5}{3}$ is less than $1\frac{3}{4}$."
			4.1.3.3	Use fraction models to add and subtract fractions with like denominators in real-world and mathematical situations. Develop a rule for addition and subtraction of fractions with like denominators.

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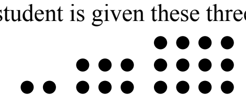
4	Number & Operation	Represent and compare fractions and decimals in real-world and mathematical situations; use place value to understand how decimals represent quantities.	4.1.3.4	Read and write decimals with words and symbols; use place value to describe decimals in terms of groups of thousands, hundreds, tens, ones, tenths, hundredths and thousandths. *For example*: Writing 362.45 is a shorter way of writing the sum: 3 hundreds + 6 tens + 2 ones + 4 tenths + 5 hundredths, which can also be written as: three hundred sixty-two and forty-five hundredths.
			4.1.3.5	Compare and order decimals and whole numbers using place value, a number line and models such as grids and base 10 blocks.
			4.1.3.6	Locate the relative position of fractions, mixed numbers and decimals on a number line.
			4.1.3.7	Read and write tenths and hundredths in decimal and fraction notations using words and symbols; know the fraction and decimal equivalents for halves and fourths. *For example*: $\frac{1}{2} = 0.5 = 0.50$ and $\frac{7}{4} = 1\frac{3}{4} = 1.75$, which can also be written as one and three-fourths or one and seventy-five hundredths.
			4.1.3.8	Round decimal values to the nearest tenth. *For example*: The number 0.36 rounded to the nearest tenth is 0.4.
	Algebra	Use input-output rules, tables and charts to represent patterns and relationships and to solve real-world and mathematical problems.	4.2.1.1	Create and use input-output rules involving addition, subtraction, multiplication and division to solve problems in various contexts. Record the inputs and outputs in a chart or table. *For example*: If the rule is "multiply by 3 and add 4," record the outputs for given inputs in a table. *Another example*: A student is given these three arrangements of dots:  Identify a pattern that is consistent with these figures, create an input-output rule that describes the pattern, and use the rule to find the number of dots in the 10th figure.

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4	Algebra	Use number sentences involving multiplication, division and unknowns to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.	4.2.2.1	Understand how to interpret number sentences involving multiplication, division and unknowns. Use real-world situations involving division to represent number sentences. *For example*: The number sentence $a \times b = 60$ can be represented by the situation in which chairs are being arranged in equal rows and the total number of chairs is 60.
			4.2.2.2	Use multiplication, division and unknowns to represent a given problem situation using a number sentence. Use number sense, properties of multiplication, and the relationship between multiplication and division to find values for the unknowns that make the number sentences true. *For example*: If $84 is to be shared equally among a group of children, the amount of money each child receives can be determined using the number sentence $84 \div n = d$. *Another example*: Find values of the unknowns or variables that make each number sentence true: $12 \times m = 36$ $s = 256 \div t$.
	Geometry & Measurement	Name, describe, classify and sketch polygons.	4.3.1.1	Describe, classify and sketch triangles, including equilateral, right, obtuse and acute triangles. Recognize triangles in various contexts.
			4.3.1.2	Describe, classify and draw quadrilaterals, including squares, rectangles, trapezoids, rhombuses, parallelograms and kites. Recognize quadrilaterals in various contexts.
		Understand angle and area as measurable attributes of real-world and mathematical objects. Use various tools to measure angles and areas.	4.3.2.1	Measure angles in geometric figures and real-world objects with a protractor or angle ruler.
			4.3.2.2	Compare angles according to size. Classify angles as acute, right and obtuse. *For example*: Compare different hockey sticks according to the angle between the blade and the shaft.
			4.3.2.3	Understand that the area of a two-dimensional figure can be found by counting the total number of same size square units that cover a shape without gaps or overlaps. Justify why length and width are multiplied to find the area of a rectangle by breaking the rectangle into one unit by one unit squares and viewing these as grouped into rows and columns. *For example*: How many copies of a square sheet of paper are needed to cover the classroom door? Measure the length and width of the door to the nearest inch and compute the area of the door.
			4.3.2.4	Find the areas of geometric figures and real-world objects that can be divided into rectangular shapes. Use square units to label area measurements.

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4	Geometry & Measurement	Use translations, reflections and rotations to establish congruency and understand symmetries.	4.3.3.1	Apply translations (slides) to figures.
			4.3.3.2	Apply reflections (flips) to figures by reflecting over vertical or horizontal lines and relate reflections to lines of symmetry.
			4.3.3.3	Apply rotations (turns) of 90˚ clockwise or counterclockwise.
			4.3.3.4	Recognize that translations, reflections and rotations preserve congruency and use them to show that two figures are congruent.
	Data Analysis	Collect, organize, display and interpret data, including data collected over a period of time and data represented by fractions and decimals.	4.4.1.1	Use tables, bar graphs, timelines and Venn diagrams to display data sets. The data may include fractions or decimals. Understand that spreadsheet tables and graphs can be used to display data.
5	Number & Operation	Divide multi-digit numbers; solve real-world and mathematical problems using arithmetic.	5.1.1.1	Divide multi-digit numbers, using efficient and generalizable procedures, based on knowledge of place value, including standard algorithms. Recognize that quotients can be represented in a variety of ways, including a whole number with a remainder, a fraction or mixed number, or a decimal. *For example*: Dividing 153 by 7 can be used to convert the improper fraction $\frac{153}{7}$ to the mixed number $21\frac{6}{7}$.
			5.1.1.2	Consider the context in which a problem is situated to select the most useful form of the quotient for the solution and use the context to interpret the quotient appropriately. *For example*: If 77 amusement ride tickets are to be distributed evenly among 4 children, each child will receive 19 tickets, and there will be one left over. If $77 is to be distributed evenly among 4 children, each will receive $19.25, with nothing left over.
			5.1.1.3	Estimate solutions to arithmetic problems in order to assess the reasonableness of results of calculations.
			5.1.1.4	Solve real-world and mathematical problems requiring addition, subtraction, multiplication and division of multi-digit whole numbers. Use various strategies, including the use of a calculator and the inverse relationships between operations, to check for accuracy. *For example*: The calculation 117 ÷ 9 = 13 can be checked by multiplying 9 and 13.

	Strand	Standard	No.	Benchmark
5	Number & Operation	Read, write, represent and compare fractions and decimals; recognize and write equivalent fractions; convert between fractions and decimals; use fractions and decimals in real-world and mathematical situations.	5.1.2.1	Read and write decimals using place value to describe decimals in terms of groups from millionths to millions. *For example*: Possible names for the number 0.37 are: 37 hundredths 3 tenths + 7 hundredths; possible names for the number 1.5 are: one and five tenths 15 tenths.
			5.1.2.2	Find 0.1 more than a number and 0.1 less than a number. Find 0.01 more than a number and 0.01 less than a number. Find 0.001 more than a number and 0.001 less than a number.
			5.1.2.3	Order fractions and decimals, including mixed numbers and improper fractions, and locate on a number line. *For example*: Which is larger 1.25 or $\frac{6}{5}$? *Another example*: In order to work properly, a part must fit through a 0.24 inch wide space. If a part is $\frac{1}{4}$ inch wide, will it fit?
			5.1.2.4	Recognize and generate equivalent decimals, fractions, mixed numbers and improper fractions in various contexts. *For example*: When comparing 1.5 and $\frac{19}{12}$, note that $1.5 = 1\frac{1}{2} = 1\frac{6}{12} = \frac{18}{12}$, so $1.5 < \frac{19}{12}$.
			5.1.2.5	Round numbers to the nearest 0.1, 0.01 and 0.001. *For example*: Fifth grade students used a calculator to find the mean of the monthly allowance in their class. The calculator display shows 25.80645161. Round this number to the nearest cent.
		Add and subtract fractions, mixed numbers and decimals to solve real-world and mathematical problems.	5.1.3.1	Add and subtract decimals and fractions, using efficient and generalizable procedures, including standard algorithms.
			5.1.3.2	Model addition and subtraction of fractions and decimals using a variety of representations. *For example*: Represent $\frac{2}{3}+\frac{1}{4}$ and $\frac{2}{3}-\frac{1}{4}$ by drawing a rectangle divided into 4 columns and 3 rows and shading the appropriate parts or by using fraction circles or bars.
			5.1.3.3	Estimate sums and differences of decimals and fractions to assess the reasonableness of results in calculations. *For example*: Recognize that $12\frac{2}{5}-3\frac{3}{4}$ is between 8 and 9 (since $\frac{2}{5}<\frac{3}{4}$).
			5.1.3.4	Solve real-world and mathematical problems requiring addition and subtraction of decimals, fractions and mixed numbers, including those involving measurement, geometry and data. *For example*: Calculate the perimeter of the soccer field when the length is 109.7 meters and the width is 73.1 meters.

	Strand	Standard	No.	Benchmark
5	Algebra	Recognize and represent patterns of change; use patterns, tables, graphs and rules to solve real-world and mathematical problems.	5.2.1.1	Create and use rules, tables, spreadsheets and graphs to describe patterns of change and solve problems. *For example*: An end-of-the-year party for 5th grade costs $100 to rent the room and $4.50 for each student. Know how to use a spreadsheet to create an input-output table that records the total cost of the party for any number of students between 90 and 150.
			5.2.1.2	Use a rule or table to represent ordered pairs of positive integers and graph these ordered pairs on a coordinate system.
		Use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving whole numbers.	5.2.2.1	Apply the commutative, associative and distributive properties and order of operations to generate equivalent numerical expressions and to solve problems involving whole numbers. *For example*: Purchase 5 pencils at 19 cents and 7 erasers at 19 cents. The numerical expression is $5 \times 19 + 7 \times 19$ which is the same as $(5 + 7) \times 19$.
		Understand and interpret equations and inequalities involving variables and whole numbers, and use them to represent and solve real-world and mathematical problems.	5.2.3.1	Determine whether an equation or inequality involving a variable is true or false for a given value of the variable. *For example*: Determine whether the inequality $1.5 + x < 10$ is true for $x = 2.8$, $x = 8.1$, or $x = 9.2$.
			5.2.3.2	Represent real-world situations using equations and inequalities involving variables. Create real-world situations corresponding to equations and inequalities. *For example*: $250 - 27 \times a = b$ can be used to represent the number of sheets of paper remaining from a packet of 250 when each student in a class of 27 is given a certain number of sheets.
			5.2.3.3	Evaluate expressions and solve equations involving variables when values for the variables are given. *For example*: Using the formula, $A= \ell w$, determine the area when the length is 5, and the width 6, and find the length when the area is 24 and the width is 4.
	Geometry & Measurement	Describe, classify, and draw representations of three-dimensional figures.	5.3.1.1	Describe and classify three-dimensional figures including cubes, prisms and pyramids by the number of edges, faces or vertices as well as the types of faces.
			5.3.1.2	Recognize and draw a net for a three-dimensional figure.

	Strand	Standard	No.	Benchmark
5	Geometry & Measurement	Determine the area of triangles and quadrilaterals; determine the surface area and volume of rectangular prisms in various contexts.	5.3.2.1	Develop and use formulas to determine the area of triangles, parallelograms and figures that can be decomposed into triangles.
			5.3.2.2	Determine the surface area of a rectangular prism by applying various strategies. *For example*: Use a net or decompose the surface into rectangles.
			5.3.2.3	Understand that the volume of a three-dimensional figure can be found by counting the total number of same-size cubic units that fill a shape without gaps or overlaps. Use cubic units to label volume measurements. *For example*: Use cubes to find the volume of a small fish tank.
			5.3.2.4	Develop and use the formulas $V = \ell wh$ and $V = Bh$ to determine the volume of rectangular prisms. Justify why base area B and height h are multiplied to find the volume of a rectangular prism by breaking the prism into layers of unit cubes.
			5.3.2.5	Use various tools to measure the volume and surface area of various objects that are shaped like rectangular prisms. *For example*: Measure the surface area of a cereal box by cutting it into rectangles. *Another example*: Measure the volume of a cereal box by using a ruler to measure its height, width and length, or by filling it with cereal and then emptying the cereal into containers of known volume.
	Data Analysis	Display and interpret data; determine mean, median and range.	5.4.1.1	Know and use the definitions of the mean, median and range of a set of data. Know how to use a spreadsheet to find the mean, median and range of a data set. Understand that the mean is a "leveling out" of data. *For example*: The set of numbers 1, 1, 4, 6 has mean 3. It can be leveled by taking one unit from the 4 and three units from the 6 and adding them to the 1s, making four 3s.
			5.4.1.2	Create and analyze double-bar graphs and line graphs by applying understanding of whole numbers, fractions and decimals. Know how to create spreadsheet tables and graphs to display data.

	Strand	Standard	No.	Benchmark
6	Number & Operation	Read, write, represent and compare positive rational numbers expressed as fractions, decimals, percents and ratios; write positive integers as products of factors; use these representations in real-world and mathematical situations.	6.1.1.1	Locate positive rational numbers on a number line and plot pairs of positive rational numbers on a coordinate grid.
			6.1.1.2	Compare positive rational numbers represented in various forms. Use the symbols < and >. *For example*: $\frac{1}{2} > 0.36$.
			6.1.1.3	Understand that percent represents parts out of 100 and ratios to 100. *For example*: 75% is equivalent to the ratio 75 to 100, which is equivalent to the ratio 3 to 4.
			6.1.1.4	Determine equivalences among fractions, decimals and percents; select among these representations to solve problems. *For example*: Since $\frac{1}{10}$ is equivalent to 10%, if a woman making $25 an hour gets a 10% raise, she will make an additional $2.50 an hour, because $2.50 is $\frac{1}{10}$ of $25.
			6.1.1.5	Factor whole numbers; express a whole number as a product of prime factors with exponents. *For example*: $24 = 2^3 \times 3$.
			6.1.1.6	Determine greatest common factors and least common multiples. Use common factors and common multiples to do arithmetic with fractions and find equivalent fractions. *For example*: Factor the numerator and denominator of a fraction to determine an equivalent fraction.
			6.1.1.7	Convert between equivalent representations of positive rational numbers. *For example*: Express $\frac{10}{7}$ as $\frac{7+3}{7} = \frac{7}{7} + \frac{3}{7} = 1\frac{3}{7}$.

	Strand	Standard	No.	Benchmark
6	Number & Operation	Understand the concept of ratio and its relationship to fractions and to the multiplication and division of whole numbers. Use ratios to solve real-world and mathematical problems.	6.1.2.1	Identify and use ratios to compare quantities; understand that comparing quantities using ratios is not the same as comparing quantities using subtraction. *For example*: In a classroom with 15 boys and 10 girls, compare the numbers by subtracting (there are 5 more boys than girls) or by dividing (there are 1.5 times as many boys as girls). The comparison using division may be expressed as a ratio of boys to girls (3 to 2 or 3:2 or 1.5 to 1).
			6.1.2.2	Apply the relationship between ratios, equivalent fractions and percents to solve problems in various contexts, including those involving mixtures and concentrations. *For example*: If 5 cups of trail mix contains 2 cups of raisins, the ratio of raisins to trail mix is 2 to 5. This ratio corresponds to the fact that the raisins are $\frac{2}{5}$ of the total, or 40% of the total. And if one trail mix consists of 2 parts peanuts to 3 parts raisins, and another consists of 4 parts peanuts to 8 parts raisins, then the first mixture has a higher concentration of peanuts.
			6.1.2.3	Determine the rate for ratios of quantities with different units. *For example*: 60 miles in 3 hours is equivalent to 20 miles in one hour (20 mph).
			6.1.2.4	Use reasoning about multiplication and division to solve ratio and rate problems. *For example*: If 5 items cost $3.75, and all items are the same price, then 1 item costs 75 cents, so 12 items cost $9.00.
		Multiply and divide decimals, fractions and mixed numbers; solve real-world and mathematical problems using arithmetic with positive rational numbers.	6.1.3.1	Multiply and divide decimals and fractions, using efficient and generalizable procedures, including standard algorithms.
			6.1.3.2	Use the meanings of fractions, multiplication, division and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions. *For example*: Just as $\frac{12}{4}=3$ means $12=3\times 4$, $\frac{2}{3}\div\frac{4}{5}=\frac{5}{6}$ means $\frac{5}{6}\times\frac{4}{5}=\frac{2}{3}$.
			6.1.3.3	Calculate the percent of a number and determine what percent one number is of another number to solve problems in various contexts. *For example*: If John has $45 and spends $15, what percent of his money did he keep?
			6.1.3.4	Solve real-world and mathematical problems requiring arithmetic with decimals, fractions and mixed numbers.
			6.1.3.5	Estimate solutions to problems with whole numbers, fractions and decimals and use the estimations to assess the reasonableness of computations and of results in the context of the problem. *For example*: The sum $\frac{1}{3}+0.25$ can be estimated to be between $\frac{1}{2}$ and 1, and this estimate can be used as a check on the result of a more detailed calculation.

	Strand	Standard	No.	Benchmark
6	Algebra	Recognize and represent relationships between varying quantities; translate from one representation to another; use patterns, tables, graphs and rules to solve real-world and mathematical problems.	6.2.1.1	Understand that a variable can be used to represent a quantity that can change, often in relationship to another changing quantity. Use variables in various contexts. *For example*: If a student earns $7 an hour in a job, the amount of money earned can be represented by a variable and is related to the number of hours worked, which also can be represented by a variable.
			6.2.1.2	Represent the relationship between two varying quantities with function rules, graphs and tables; translate between any two of these representations. *For example*: Describe the terms in the sequence of perfect squares t = 1, 4, 9, 16, ... by using the rule $t = n^2$ for n = 1, 2, 3, 4, ...
		Use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving positive rational numbers.	6.2.2.1	Apply the associative, commutative and distributive properties and order of operations to generate equivalent expressions and to solve problems involving positive rational numbers. *For example*: $\frac{32}{15}\times\frac{5}{6}=\frac{32\times5}{15\times6}=\frac{2\times16\times5}{3\times5\times3\times2}=\frac{16}{9}\times\frac{2}{2}\times\frac{5}{5}=\frac{16}{9}$. *Another example*: Use the distributive law to write: $\frac{1}{2}+\frac{1}{3}\left(\frac{9}{2}-\frac{15}{8}\right)=\frac{1}{2}+\frac{1}{3}\times\frac{9}{2}-\frac{1}{3}\times\frac{15}{8}=\frac{1}{2}+\frac{3}{2}-\frac{5}{8}=2-\frac{5}{8}=1\frac{3}{8}$.
		Understand and interpret equations and inequalities involving variables and positive rational numbers. Use equations and inequalities to represent real-world and mathematical problems; use the idea of maintaining equality to solve equations. Interpret solutions in the original context.	6.2.3.1	Represent real-world or mathematical situations using equations and inequalities involving variables and positive rational numbers. *For example*: The number of miles m in a k kilometer race is represented by the equation $m = 0.62\,k$.
			6.2.3.2	Solve equations involving positive rational numbers using number sense, properties of arithmetic and the idea of maintaining equality on both sides of the equation. Interpret a solution in the original context and assess the reasonableness of results. *For example*: A cellular phone company charges $0.12 per minute. If the bill was $11.40 in April, how many minutes were used?

	Strand	Standard	No.	Benchmark
6	Geometry & Measurement	Calculate perimeter, area, surface area and volume of two- and three-dimensional figures to solve real-world and mathematical problems.	6.3.1.1	Calculate the surface area and volume of prisms and use appropriate units, such as cm^2 and cm^3. Justify the formulas used. Justification may involve decomposition, nets or other models. *For example*: The surface area of a triangular prism can be derived by decomposing the surface into two triangles and three rectangles.
			6.3.1.2	Calculate the area of quadrilaterals. Quadrilaterals include squares, rectangles, rhombuses, parallelograms, trapezoids and kites. When formulas are used, be able to explain why they are valid. *For example*: The area of a kite is one-half the product of the lengths of the diagonals, and this can be justified by decomposing the kite into two triangles.
			6.3.1.3	Estimate the perimeter and area of irregular figures on a grid when they cannot be decomposed into common figures and use correct units, such as cm and cm^2.
		Understand and use relationships between angles in geometric figures.	6.3.2.1	Solve problems using the relationships between the angles formed by intersecting lines. *For example*: If two streets cross, forming four corners such that one of the corners forms an angle of 120˚, determine the measures of the remaining three angles. *Another example*: Recognize that pairs of interior and exterior angles in polygons have measures that sum to 180˚.
			6.3.2.2	Determine missing angle measures in a triangle using the fact that the sum of the interior angles of a triangle is 180˚. Use models of triangles to illustrate this fact. *For example*: Cut a triangle out of paper, tear off the corners and rearrange these corners to form a straight line. *Another example*: Recognize that the measures of the two acute angles in a right triangle sum to 90˚.
			6.3.2.3	Develop and use formulas for the sums of the interior angles of polygons by decomposing them into triangles.
		Choose appropriate units of measurement and use ratios to convert within measurement systems to solve real-world and mathematical problems.	6.3.3.1	Solve problems in various contexts involving conversion of weights, capacities, geometric measurements and times within measurement systems using appropriate units.
			6.3.3.2	Estimate weights, capacities and geometric measurements using benchmarks in measurement systems with appropriate units. *For example*: Estimate the height of a house by comparing to a 6-foot man standing nearby.

	Strand	Standard	No.	Benchmark
6	Data Analysis & Probability	Use probabilities to solve real-world and mathematical problems; represent probabilities using fractions, decimals and percents.	6.4.1.1	Determine the sample space (set of possible outcomes) for a given experiment and determine which members of the sample space are related to certain events. Sample space may be determined by the use of tree diagrams, tables or pictorial representations. *For example*: A 6×6 table with entries such as (1,1), (1,2), (1,3), …, (6,6) can be used to represent the sample space for the experiment of simultaneously rolling two number cubes.
			6.4.1.2	Determine the probability of an event using the ratio between the size of the event and the size of the sample space; represent probabilities as percents, fractions and decimals between 0 and 1 inclusive. Understand that probabilities measure likelihood. *For example*: Each outcome for a balanced number cube has probability $\frac{1}{6}$, and the probability of rolling an even number is $\frac{1}{2}$.
			6.4.1.3	Perform experiments for situations in which the probabilities are known, compare the resulting relative frequencies with the known probabilities; know that there may be differences. *For example*: Heads and tails are equally likely when flipping a fair coin, but if several different students flipped fair coins 10 times, it is likely that they will find a variety of relative frequencies of heads and tails.
			6.4.1.4	Calculate experimental probabilities from experiments; represent them as percents, fractions and decimals between 0 and 1 inclusive. Use experimental probabilities to make predictions when actual probabilities are unknown. *For example*: Repeatedly draw colored chips with replacement from a bag with an unknown mixture of chips, record relative frequencies, and use the results to make predictions about the contents of the bag.
7	Number & Operation	Read, write, represent and compare positive and negative rational numbers, expressed as integers, fractions and decimals.	7.1.1.1	Know that every rational number can be written as the ratio of two integers or as a terminating or repeating decimal. Recognize that π is not rational, but that it can be approximated by rational numbers such as $\frac{22}{7}$ and 3.14.
			7.1.1.2	Understand that division of two integers will always result in a rational number. Use this information to interpret the decimal result of a division problem when using a calculator. *For example*: $\frac{125}{30}$ gives 4.16666667 on a calculator. This answer is not exact. The exact answer can be expressed as $4\frac{1}{6}$, which is the same as $4.1\overline{6}$. The calculator expression does not guarantee that the 6 is repeated, but that possibility should be anticipated.
			7.1.1.3	Locate positive and negative rational numbers on the number line, understand the concept of opposites, and plot pairs of positive and negative rational numbers on a coordinate grid.

	Strand	Standard	No.	Benchmark
7	Number & Operation	Read, write, represent and compare positive and negative rational numbers, expressed as integers, fractions and decimals.	7.1.1.4	Compare positive and negative rational numbers expressed in various forms using the symbols <, >, ≤, ≥. *For example*: $-\frac{1}{2} < -0.36$.
			7.1.1.5	Recognize and generate equivalent representations of positive and negative rational numbers, including equivalent fractions. *For example*: $-\frac{40}{12} = -\frac{120}{36} = -\frac{10}{3} = -3.\overline{3}$.
		Calculate with positive and negative rational numbers, and rational numbers with whole number exponents, to solve real-world and mathematical problems.	7.1.2.1	Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents. *For example*: $3^4 \times \left(\frac{1}{2}\right)^2 = \frac{81}{4}$.
			7.1.2.2	Use real-world contexts and the inverse relationship between addition and subtraction to explain why the procedures of arithmetic with negative rational numbers make sense. *For example*: Multiplying a distance by -1 can be thought of as representing that same distance in the opposite direction. Multiplying by -1 a second time reverses directions again, giving the distance in the original direction.
			7.1.2.3	Understand that calculators and other computing technologies often truncate or round numbers. *For example*: A decimal that repeats or terminates after a large number of digits is truncated or rounded.
			7.1.2.4	Solve problems in various contexts involving calculations with positive and negative rational numbers and positive integer exponents, including computing simple and compound interest.
			7.1.2.5	Use proportional reasoning to solve problems involving ratios in various contexts. *For example*: A recipe calls for milk, flour and sugar in a ratio of 4:6:3 (this is how recipes are often given in large institutions, such as hospitals). How much flour and milk would be needed with 1 cup of sugar?
			7.1.2.6	Demonstrate an understanding of the relationship between the absolute value of a rational number and distance on a number line. Use the symbol for absolute value. *For example*: $\lvert -3 \rvert$ represents the distance from -3 to 0 on a number line or 3 units; the distance between 3 and $\frac{9}{2}$ on the number line is $\lvert 3 - \frac{9}{2} \rvert$ or $\frac{3}{2}$.

	Strand	Standard	No.	Benchmark
7	Algebra	Understand the concept of proportionality in real-world and mathematical situations, and distinguish between proportional and other relationships.	7.2.1.1	Understand that a relationship between two variables, *x* and *y*, is proportional if it can be expressed in the form $\frac{y}{x}=k$ or $y=kx$. Distinguish proportional relationships from other relationships, including inversely proportional relationships ($xy=k$ or $y=\frac{k}{x}$). *For example*: The radius and circumference of a circle are proportional, whereas the length *x* and the width *y* of a rectangle with area 12 are inversely proportional, since *xy* = 12 or equivalently, $y=\frac{12}{x}$.
			7.2.1.2	Understand that the graph of a proportional relationship is a line through the origin whose slope is the unit rate (constant of proportionality). Know how to use graphing technology to examine what happens to a line when the unit rate is changed.
		Recognize proportional relationships in real-world and mathematical situations; represent these and other relationships with tables, verbal descriptions, symbols and graphs; solve problems involving proportional relationships and explain results in the original context.	7.2.2.1	Represent proportional relationships with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another. Determine the unit rate (constant of proportionality or slope) given any of these representations. *For example*: Larry drives 114 miles and uses 5 gallons of gasoline. Sue drives 300 miles and uses 11.5 gallons of gasoline. Use equations and graphs to compare fuel efficiency and to determine the costs of various trips.
			7.2.2.2	Solve multi-step problems involving proportional relationships in numerous contexts. *For example*: Distance-time, percent increase or decrease, discounts, tips, unit pricing, lengths in similar geometric figures, and unit conversion when a conversion factor is given, including conversion between different measurement systems. *Another example*: How many kilometers are there in 26.2 miles?
			7.2.2.3	Use knowledge of proportions to assess the reasonableness of solutions. *For example*: Recognize that it would be unreasonable for a cashier to request $200 if you purchase a $225 item at 25% off.
			7.2.2.4	Represent real-world or mathematical situations using equations and inequalities involving variables and positive and negative rational numbers. *For example*: "Four-fifths is three greater than the opposite of a number" can be represented as $\frac{4}{5}=-n+3$, and "height no bigger than half the radius" can be represented as $h\le\frac{r}{2}$. *Another example*: "*x* is at least -3 and less than 5" can be represented as $-3\le x<5$, and also on a number line.

	Strand	Standard	No.	Benchmark
7	Algebra	Apply understanding of order of operations and algebraic properties to generate equivalent numerical and algebraic expressions containing positive and negative rational numbers and grouping symbols; evaluate such expressions.	7.2.3.1	Generate equivalent numerical and algebraic expressions containing rational numbers and whole number exponents. Properties of algebra include associative, commutative and distributive laws. *For example*: Combine like terms (use the distributive law) to write $3x-7x+1=(3-7)x+1=-4x+1$.
			7.2.3.2	Evaluate algebraic expressions containing rational numbers and whole number exponents at specified values of their variables. *For example*: Evaluate the expression $\frac{1}{3}(2x-5)^2$ at $x = 5$.
			7.2.3.3	Apply understanding of order of operations and grouping symbols when using calculators and other technologies. *For example*: Recognize the conventions of using a carat (^ raise to a power), asterisk (* multiply), and also pay careful attention to the use of nested parentheses.
		Represent real-world and mathematical situations using equations with variables. Solve equations symbolically, using the properties of equality. Also solve equations graphically and numerically. Interpret solutions in the original context.	7.2.4.1	Represent relationships in various contexts with equations involving variables and positive and negative rational numbers. Use the properties of equality to solve for the value of a variable. Interpret the solution in the original context. *For example*: Solve for w in the equation $P = 2w + 2\ell$ when $P = 3.5$ and $\ell = 0.4$. *Another example*: To post an Internet website, Mary must pay $300 for initial set up and a monthly fee of $12. She has $842 in savings, how long can she sustain her website?
			7.2.4.2	Solve equations resulting from proportional relationships in various contexts. *For example*: Given the side lengths of one triangle and one side length of a second triangle that is similar to the first, find the remaining side lengths of the second triangle. *Another example*: Determine the price of 12 yards of ribbon if 5 yards of ribbon cost $1.85.

	Strand	Standard	No.	Benchmark
7	Geometry & Measurement	Use reasoning with proportions and ratios to determine measurements, justify formulas and solve real-world and mathematical problems involving circles and related geometric figures.	7.3.1.1	Demonstrate an understanding of the proportional relationship between the diameter and circumference of a circle and that the unit rate (constant of proportionality) is π . Calculate the circumference and area of circles and sectors of circles to solve problems in various contexts.
			7.3.1.2	Calculate the volume and surface area of cylinders and justify the formulas used. *For example*: Justify the formula for the surface area of a cylinder by decomposing the surface into two circles and a rectangle.
		Analyze the effect of change of scale, translations and reflections on the attributes of two-dimensional figures.	7.3.2.1	Describe the properties of similarity, compare geometric figures for similarity, and determine scale factors. *For example*: Corresponding angles in similar geometric figures have the same measure.
			7.3.2.2	Apply scale factors, length ratios and area ratios to determine side lengths and areas of similar geometric figures. *For example*: If two similar rectangles have heights of 3 and 5, and the first rectangle has a base of length 7, the base of the second rectangle has length $\frac{35}{3}$.
			7.3.2.3	Use proportions and ratios to solve problems involving scale drawings and conversions of measurement units. *For example*: 1 square foot equals 144 square inches. *Another example:* In a map where 1 inch represents 50 miles, $\frac{1}{2}$ inch represents 25 miles.
			7.3.2.4	Graph and describe translations and reflections of figures on a coordinate grid and determine the coordinates of the vertices of the figure after the transformation. *For example*: The point (1, 2) moves to (-1, 2) after reflection about the *y*-axis.
	Data Analysis & Probability	Use mean, median and range to draw conclusions about data and make predictions.	7.4.1.1	Determine mean, median and range for quantitative data and from data represented in a display. Use these quantities to draw conclusions about the data, compare different data sets, and make predictions. *For example*: By looking at data from the past, Sandy calculated that the mean gas mileage for her car was 28 miles per gallon. She expects to travel 400 miles during the next week. Predict the approximate number of gallons that she will use.
			7.4.1.2	Describe the impact that inserting or deleting a data point has on the mean and the median of a data set. Know how to create data displays using a spreadsheet to examine this impact. *For example*: How does dropping the lowest test score affect a student's mean test score?

	Strand	Standard	No.	Benchmark
7	Data Analysis & Probability	Display and interpret data in a variety of ways, including circle graphs and histograms.	7.4.2.1	Use reasoning with proportions to display and interpret data in circle graphs (pie charts) and histograms. Choose the appropriate data display and know how to create the display using a spreadsheet or other graphing technology.
		Calculate probabilities and reason about probabilities using proportions to solve real-world and mathematical problems.	7.4.3.1	Use random numbers generated by a calculator or a spreadsheet or taken from a table to simulate situations involving randomness, make a histogram to display the results, and compare the results to known probabilities. *For example*: Use a spreadsheet function such as RANDBETWEEN(1, 10) to generate random whole numbers from 1 to 10, and display the results in a histogram.
			7.4.3.2	Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions. *For example*: Determine probabilities for different outcomes in game spinners by finding fractions of the area of the spinner.
			7.4.3.3	Use proportional reasoning to draw conclusions about and predict relative frequencies of outcomes based on probabilities. *For example*: When rolling a number cube 600 times, one would predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
8	Number & Operation	Read, write, compare, classify and represent real numbers, and use them to solve problems in various contexts.	8.1.1.1	Classify real numbers as rational or irrational. Know that when a square root of a positive integer is not an integer, then it is irrational. Know that the sum of a rational number and an irrational number is irrational, and the product of a non-zero rational number and an irrational number is irrational. *For example*: Classify the following numbers as whole numbers, integers, rational numbers, irrational numbers, recognizing that some numbers belong in more than one category: $\frac{6}{3}$, $\frac{3}{6}$, $3.\overline{6}$, $\frac{\pi}{2}$, $-\sqrt{4}$, $\sqrt{10}$, -6.7.
			8.1.1.2	Compare real numbers; locate real numbers on a number line. Identify the square root of a positive integer as an integer, or if it is not an integer, locate it as a real number between two consecutive positive integers. *For example*: Put the following numbers in order from smallest to largest: $2, \sqrt{3}, -4, -6.8, -\sqrt{37}$. *Another example*: $\sqrt{68}$ is an irrational number between 8 and 9.

	Strand	Standard	No.	Benchmark
8	Number & Operation	Read, write, compare, classify and represent real numbers, and use them to solve problems in various contexts.	8.1.1.3	Determine rational approximations for solutions to problems involving real numbers. *For example*: A calculator can be used to determine that $\sqrt{7}$ is approximately 2.65. *Another example*: To check that $1\frac{5}{12}$ is slightly bigger than $\sqrt{2}$, do the calculation $\left(1\frac{5}{12}\right)^2=\left(\frac{17}{12}\right)^2=\frac{289}{144}=2\frac{1}{144}$. *Another example*: Knowing that $\sqrt{10}$ is between 3 and 4, try squaring numbers like 3.5, 3.3, 3.1 to determine that 3.1 is a reasonable rational approximation of $\sqrt{10}$.
			8.1.1.4	Know and apply the properties of positive and negative integer exponents to generate equivalent numerical expressions. *For example*: $3^2 \times 3^{(-5)} = 3^{(-3)}\left(\frac{1}{3}\right)^3 = \frac{1}{27}$.
			8.1.1.5	Express approximations of very large and very small numbers using scientific notation; understand how calculators display numbers in scientific notation. Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation, using the correct number of significant digits when physical measurements are involved. *For example*: $(4.2\times10^4)\times(8.25\times10^3)=3.465\times10^8$, but if these numbers represent physical measurements, the answer should be expressed as 3.5×10^8 because the first factor, 4.2×10^4, only has two significant digits.
	Algebra	Understand the concept of function in real-world and mathematical situations, and distinguish between linear and non-linear functions.	8.2.1.1	Understand that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable. Use functional notation, such as $f(x)$, to represent such relationships. *For example*: The relationship between the area of a square and the side length can be expressed as $f(x)=x^2$. In this case, $f(5)=25$, which represents the fact that a square of side length 5 units has area 25 units squared.
			8.2.1.2	Use linear functions to represent relationships in which changing the input variable by some amount leads to a change in the output variable that is a constant times that amount. *For example*: Uncle Jim gave Emily $50 on the day she was born and $25 on each birthday after that. The function $f(x)=50+25x$ represents the amount of money Jim has given after x years. The rate of change is $25 per year.

	Strand	Standard	No.	Benchmark
8	Algebra	Understand the concept of function in real-world and mathematical situations, and distinguish between linear and non-linear functions.	8.2.1.3	Understand that a function is linear if it can be expressed in the form $f(x)=mx+b$ or if its graph is a straight line. *For example*: The function $f(x)=x^2$ is not a linear function because its graph contains the points (1,1), (-1,1) and (0,0), which are not on a straight line.
			8.2.1.4	Understand that an arithmetic sequence is a linear function that can be expressed in the form $f(x)=mx+b$, where $x = 0, 1, 2, 3,\ldots$. *For example*: The arithmetic sequence 3, 7, 11, 15, …, can be expressed as $f(x) = 4x + 3$.
			8.2.1.5	Understand that a geometric sequence is a non-linear function that can be expressed in the form $f(x)=ab^x$, where $x = 0, 1, 2, 3,\ldots$. *For example*: The geometric sequence 6, 12, 24, 48, … , can be expressed in the form $f(x) = 6(2^x)$.
		Recognize linear functions in real-world and mathematical situations; represent linear functions and other functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions and explain results in the original context.	8.2.2.1	Represent linear functions with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another.
			8.2.2.2	Identify graphical properties of linear functions including slopes and intercepts. Know that the slope equals the rate of change, and that the y-intercept is zero when the function represents a proportional relationship.
			8.2.2.3	Identify how coefficient changes in the equation $f(x) = mx + b$ affect the graphs of linear functions. Know how to use graphing technology to examine these effects.
			8.2.2.4	Represent arithmetic sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems. *For example*: If a girl starts with $100 in savings and adds $10 at the end of each month, she will have $100 + 10x$ dollars after x months.
			8.2.2.5	Represent geometric sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems. *For example*: If a girl invests $100 at 10% annual interest, she will have $100(1.1^x)$ dollars after x years.

	Strand	Standard	No.	Benchmark
8	Algebra	Generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions.	8.2.3.1	Evaluate algebraic expressions, including expressions containing radicals and absolute values, at specified values of their variables. *For example*: Evaluate $\pi r^2 h$ when $r = 3$ and $h = 0.5$, and then use an approximation of π, to obtain an approximate answer.
			8.2.3.2	Justify steps in generating equivalent expressions by identifying the properties used, including the properties of algebra. Properties include the associative, commutative and distributive laws, and the order of operations, including grouping symbols.
		Represent real-world and mathematical situations using equations and inequalities involving linear expressions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	8.2.4.1	Use linear equations to represent situations involving a constant rate of change, including proportional and non-proportional relationships. *For example*: For a cylinder with fixed radius of length 5, the surface area $A = 2\pi(5)h + 2\pi(5)^2 = 10\pi h + 50\pi$, is a linear function of the height h, but it is not proportional to the height.
			8.2.4.2	Solve multi-step equations in one variable. Solve for one variable in a multi-variable equation in terms of the other variables. Justify the steps by identifying the properties of equalities used. *For example*: The equation $10x + 17 = 3x$ can be changed to $7x + 17 = 0$, and then to $7x = -17$ by adding/subtracting the same quantities to both sides. These changes do not change the solution of the equation. *Another example*: Express the radius of a circle in terms of its circumference.
			8.2.4.3	Express linear equations in slope-intercept, point-slope and standard forms, and convert between these forms. Given sufficient information, find an equation of a line. *For example*: Determine an equation of the line through the points (-1,6) and (2/3, -3/4).
			8.2.4.4	Use linear inequalities to represent relationships in various contexts. *For example*: A gas station charges $0.10 less per gallon of gasoline if a customer also gets a car wash. Without the car wash, gas costs $2.79 per gallon. The car wash is $8.95. What are the possible amounts (in gallons) of gasoline that you can buy if you also get a car wash and can spend at most $35?
			8.2.4.5	Solve linear inequalities using properties of inequalities. Graph the solutions on a number line. *For example*: The inequality $-3x < 6$ is equivalent to $x > -2$, which can be represented on the number line by shading in the interval to the right of -2.

	Strand	Standard	No.	Benchmark
8	Algebra	Represent real-world and mathematical situations using equations and inequalities involving linear expressions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	8.2.4.6	Represent relationships in various contexts with equations and inequalities involving the absolute value of a linear expression. Solve such equations and inequalities and graph the solutions on a number line. *For example*: A cylindrical machine part is manufactured with a radius of 2.1 cm, with a tolerance of 1/100 cm. The radius r satisfies the inequality $\|r - 2.1\| \leq .01$.
			8.2.4.7	Represent relationships in various contexts using systems of linear equations. Solve systems of linear equations in two variables symbolically, graphically and numerically. *For example*: Marty's cell phone company charges $15 per month plus $0.04 per minute for each call. Jeannine's company charges $0.25 per minute. Use a system of equations to determine the advantages of each plan based on the number of minutes used.
			8.2.4.8	Understand that a system of linear equations may have no solution, one solution, or an infinite number of solutions. Relate the number of solutions to pairs of lines that are intersecting, parallel or identical. Check whether a pair of numbers satisfies a system of two linear equations in two unknowns by substituting the numbers into both equations.
			8.2.4.9	Use the relationship between square roots and squares of a number to solve problems. *For example*: If $\pi x^2 = 5$, then $\|x\| = \sqrt{\frac{5}{\pi}}$, or equivalently, $x = \sqrt{\frac{5}{\pi}}$ or $x = -\sqrt{\frac{5}{\pi}}$. If x is understood as the radius of a circle in this example, then the negative solution should be discarded and $x = \sqrt{\frac{5}{\pi}}$.
	Geometry & Measurement	Solve problems involving right triangles using the Pythagorean Theorem and its converse.	8.3.1.1	Use the Pythagorean Theorem to solve problems involving right triangles. *For example*: Determine the perimeter of a right triangle, given the lengths of two of its sides. *Another example*: Show that a triangle with side lengths 4, 5 and 6 is not a right triangle.
			8.3.1.2	Determine the distance between two points on a horizontal or vertical line in a coordinate system. Use the Pythagorean Theorem to find the distance between any two points in a coordinate system.
			8.3.1.3	Informally justify the Pythagorean Theorem by using measurements, diagrams and computer software.
		Solve problems involving parallel and perpendicular lines on a coordinate system.	8.3.2.1	Understand and apply the relationships between the slopes of parallel lines and between the slopes of perpendicular lines. Dynamic graphing software may be used to examine the relationships between lines and their equations.

	Strand	Standard	No.	Benchmark
8	Geometry & Measurement	Solve problems involving parallel and perpendicular lines on a coordinate system.	8.3.2.2	Analyze polygons on a coordinate system by determining the slopes of their sides. *For example*: Given the coordinates of four points, determine whether the corresponding quadrilateral is a parallelogram.
			8.3.2.3	Given a line on a coordinate system and the coordinates of a point not on the line, find lines through that point that are parallel and perpendicular to the given line, symbolically and graphically.
	Data Analysis & Probability	Interpret data using scatterplots and approximate lines of best fit. Use lines of best fit to draw conclusions about data.	8.4.1.1	Collect, display and interpret data using scatterplots. Use the shape of the scatterplot to informally estimate a line of best fit and determine an equation for the line. Use appropriate titles, labels and units. Know how to use graphing technology to display scatterplots and corresponding lines of best fit.
			8.4.1.2	Use a line of best fit to make statements about approximate rate of change and to make predictions about values not in the original data set. *For example*: Given a scatterplot relating student heights to shoe sizes, predict the shoe size of a 5'4" student, even if the data does not contain information for a student of that height.
			8.4.1.3	Assess the reasonableness of predictions using scatterplots by interpreting them in the original context. *For example*: A set of data may show that the number of women in the U.S. Senate is growing at a certain rate each election cycle. Is it reasonable to use this trend to predict the year in which the Senate will eventually include 1000 female Senators?
9, 10, 11	Algebra	Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods.	9.2.1.1	Understand the definition of a function. Use functional notation and evaluate a function at a given point in its domain. *For example*: If $f(x)=\frac{1}{x^2-3}$, find $f(-4)$.
			9.2.1.2	Distinguish between functions and other relations defined symbolically, graphically or in tabular form.
			9.2.1.3	Find the domain of a function defined symbolically, graphically or in a real-world context. *For example*: The formula $f(x) = \pi x^2$ can represent a function whose domain is all real numbers, but in the context of the area of a circle, the domain would be restricted to positive x.
			9.2.1.4	Obtain information and draw conclusions from graphs of functions and other relations. *For example*: If a graph shows the relationship between the elapsed flight time of a golf ball at a given moment and its height at that same moment, identify the time interval during which the ball is at least 100 feet above the ground.

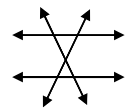
	Strand	Standard	No.	Benchmark
9, 10, 11	Algebra	Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate.	9.2.1.5	Identify the vertex, line of symmetry and intercepts of the parabola corresponding to a quadratic function, using symbolic and graphical methods, when the function is expressed in the form $f(x) = ax^2 + bx + c$, in the form $f(x) = a(x - \mathrm{h})^2 + k$, or in factored form.
			9.2.1.6	Identify intercepts, zeros, maxima, minima and intervals of increase and decrease from the graph of a function.
			9.2.1.7	Understand the concept of an asymptote and identify asymptotes for exponential functions and reciprocals of linear functions, using symbolic and graphical methods.
			9.2.1.8	Make qualitative statements about the rate of change of a function, based on its graph or table of values. *For example*: The function $f(x) = 3^x$ increases for all x, but it increases faster when $x > 2$ than it does when $x < 2$.
			9.2.1.9	Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations. *For example*: Determine how the graph of $f(x) = \|x - h\| + k$ changes as h and k change.
		Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.	9.2.2.1	Represent and solve problems in various contexts using linear and quadratic functions. *For example*: Write a function that represents the area of a rectangular garden that can be surrounded with 32 feet of fencing, and use the function to determine the possible dimensions of such a garden if the area must be at least 50 square feet.
			9.2.2.2	Represent and solve problems in various contexts using exponential functions, such as investment growth, depreciation and population growth.
			9.2.2.3	Sketch graphs of linear, quadratic and exponential functions, and translate between graphs, tables and symbolic representations. Know how to use graphing technology to graph these functions.
			9.2.2.4	Express the terms in a geometric sequence recursively and by giving an explicit (closed form) formula, and express the partial sums of a geometric series recursively. *For example*: A closed form formula for the terms t_n in the geometric sequence 3, 6, 12, 24, ... is $t_n = 3(2)^{n-1}$, where $n = 1, 2, 3, \ldots$, and this sequence can be expressed recursively by writing $t_1 = 3$ and $t_n = 2t_{n-1}$, for $n \geq 2$. *Another example*: the partial sums s_n of the series 3 + 6 + 12 + 24 + ... can be expressed recursively by writing $s_1 = 3$ and $s_n = 3 + 2s_{n-1}$, for $n \geq 2$.

	Strand	Standard	No.	Benchmark
9, 10, 11	Algebra	Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.	9.2.2.5	Recognize and solve problems that can be modeled using finite geometric sequences and series, such as home mortgage and other compound interest examples. Know how to use spreadsheets and calculators to explore geometric sequences and series in various contexts.
			9.2.2.6	Sketch the graphs of common non-linear functions such as $f(x)=\sqrt{x}$, $f(x)=\lvert x\rvert$, $f(x)=\frac{1}{x}$, $f(x) = x^3$, and translations of these functions, such as $f(x)=\sqrt{x-2}+4$. Know how to use graphing technology to graph these functions.
		Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.	9.2.3.1	Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified points in their domains.
			9.2.3.2	Add, subtract and multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.
			9.2.3.3	Factor common monomial factors from polynomials, factor quadratic polynomials, and factor the difference of two squares. *For example*: $9x^6 - x^4 = (3x^3 - x^2)(3x^3 + x^2)$.
			9.2.3.4	Add, subtract, multiply, divide and simplify algebraic fractions. *For example*: $\frac{1}{1-x}+\frac{x}{1+x}$ is equivalent to $\frac{1+2x-x^2}{1-x^2}$.
			9.2.3.5	Check whether a given complex number is a solution of a quadratic equation by substituting it for the variable and evaluating the expression, using arithmetic with complex numbers. *For example*: The complex number $\frac{1+i}{2}$ is a solution of $2x^2 - 2x + 1 = 0$, since $2\left(\frac{1+i}{2}\right)^2-2\left(\frac{1+i}{2}\right)+1=i-(1+i)+1=0$.

	Strand	Standard	No.	Benchmark
9, 10, 11	Algebra	Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.	9.2.3.6	Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions, including those involving n^{th} roots. *For example*: $\sqrt{2}\times\sqrt{7}=2^{\frac{1}{2}}\times 7^{\frac{1}{2}}=14^{\frac{1}{2}}=\sqrt{14}$. Rules for computing directly with radicals may also be used: $\sqrt{2}\times\sqrt{x}=\sqrt{2x}$.
			9.2.3.7	Justify steps in generating equivalent expressions by identifying the properties used. Use substitution to check the equality of expressions for some particular values of the variables; recognize that checking with substitution does not guarantee equality of expressions for all values of the variables.
		Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential, and nth root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	9.2.4.1	Represent relationships in various contexts using quadratic equations and inequalities. Solve quadratic equations and inequalities by appropriate methods including factoring, completing the square, graphing and the quadratic formula. Find non-real complex roots when they exist. Recognize that a particular solution may not be applicable in the original context. Know how to use calculators, graphing utilities or other technology to solve quadratic equations and inequalities. *For example*: A diver jumps from a 20 meter platform with an upward velocity of 3 meters per second. In finding the time at which the diver hits the surface of the water, the resulting quadratic equation has a positive and a negative solution. The negative solution should be discarded because of the context.
			9.2.4.2	Represent relationships in various contexts using equations involving exponential functions; solve these equations graphically or numerically. Know how to use calculators, graphing utilities or other technology to solve these equations.
			9.2.4.3	Recognize that to solve certain equations, number systems need to be extended from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers, and from real numbers to complex numbers. In particular, non-real complex numbers are needed to solve some quadratic equations with real coefficients.
			9.2.4.4	Represent relationships in various contexts using systems of linear inequalities; solve them graphically. Indicate which parts of the boundary are included in and excluded from the solution set using solid and dotted lines.
			9.2.4.5	Solve linear programming problems in two variables using graphical methods.

	Strand	Standard	No.	Benchmark
9, 10, 11	Algebra	Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential and nth root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	9.2.4.6	Represent relationships in various contexts using absolute value inequalities in two variables; solve them graphically. *For example*: If a pipe is to be cut to a length of 5 meters accurate to within a tenth of its diameter, the relationship between the length x of the pipe and its diameter y satisfies the inequality $\mid x - 5 \mid \leq 0.1y$.
			9.2.4.7	Solve equations that contain radical expressions. Recognize that extraneous solutions may arise when using symbolic methods. *For example*: The equation $\sqrt{x-9} = 9\sqrt{x}$ may be solved by squaring both sides to obtain $x - 9 = 81x$, which has the solution $x = -\frac{9}{80}$. However, this is not a solution of the original equation, so it is an extraneous solution that should be discarded. The original equation has no solution in this case. *Another example*: Solve $\sqrt[3]{-x+1} = -5$.
			9.2.4.8	Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.
	Geometry & Measurement	Calculate measurements of plane and solid geometric figures; know that physical measurements depend on the choice of a unit and that they are approximations.	9.3.1.1	Determine the surface area and volume of pyramids, cones and spheres. Use measuring devices or formulas as appropriate. *For example*: Measure the height and radius of a cone and then use a formula to find its volume.
			9.3.1.2	Compose and decompose two- and three-dimensional figures; use decomposition to determine the perimeter, area, surface area and volume of various figures. *For example*: Find the volume of a regular hexagonal prism by decomposing it into six equal triangular prisms.
			9.3.1.3	Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems. *For example*: 60 miles/hour = 60 miles/hour × 5280 feet/mile × 1 hour/3600 seconds = 88 feet/second.
			9.3.1.4	Understand and apply the fact that the effect of a scale factor k on length, area and volume is to multiply each by k, k^2 and k^3, respectively.

	Strand	Standard	No.	Benchmark
9, 10, 11	Geometry & Measurement	Calculate measurements of plane and solid geometric figures; know that physical measurements depend on the choice of a unit and that they are approximations.	9.3.1.5	Make reasonable estimates and judgments about the accuracy of values resulting from calculations involving measurements. *For example*: Suppose the sides of a rectangle are measured to the nearest tenth of a centimeter at 2.6 cm and 9.8 cm. Because of measurement errors, the width could be as small as 2.55 cm or as large as 2.65 cm, with similar errors for the height. These errors affect calculations. For instance, the actual area of the rectangle could be smaller than 25 cm^2 or larger than 26 cm^2, even though $2.6 \times 9.8 = 25.48$.
		Construct logical arguments, based on axioms, definitions and theorems, to prove theorems and other results in geometry.	9.3.2.1	Understand the roles of axioms, definitions, undefined terms and theorems in logical arguments.
			9.3.2.2	Accurately interpret and use words and phrases in geometric proofs such as "if…then," "if and only if," "all," and "not." Recognize the logical relationships between an "if…then" statement and its inverse, converse and contrapositive. *For example*: The statement "If you don't do your homework, you can't go to the dance" is not logically equivalent to its inverse "If you do your homework, you can go to the dance."
			9.3.2.3	Assess the validity of a logical argument and give counterexamples to disprove a statement.
			9.3.2.4	Construct logical arguments and write proofs of theorems and other results in geometry, including proofs by contradiction. Express proofs in a form that clearly justifies the reasoning, such as two-column proofs, paragraph proofs, flow charts or illustrations. *For example*: Prove that the sum of the interior angles of a pentagon is 540˚ using the fact that the sum of the interior angles of a triangle is 180˚.
			9.3.2.5	Use technology tools to examine theorems, test conjectures, perform constructions and develop mathematical reasoning skills in multi-step problems. The tools may include compass and straight edge, dynamic geometry software, design software or Internet applets.
		Know and apply properties of geometric figures to solve real-world and mathematical problems and to logically justify results in geometry.	9.3.3.1	Know and apply properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve problems and logically justify results. *For example*: Prove that the perpendicular bisector of a line segment is the set of all points equidistant from the two endpoints, and use this fact to solve problems and justify other results.

	Strand	Standard	No.	Benchmark
9, 10, 11	Geometry & Measurement	Know and apply properties of geometric figures to solve real-world and mathematical problems and to logically justify results in geometry.	9.3.3.2	Know and apply properties of angles, including corresponding, exterior, interior, vertical, complementary and supplementary angles, to solve problems and logically justify results. *For example*: Prove that two triangles formed by a pair of intersecting lines and a pair of parallel lines (an "X" trapped between two parallel lines) are similar. 
			9.3.3.3	Know and apply properties of equilateral, isosceles and scalene triangles to solve problems and logically justify results. *For example*: Use the triangle inequality to prove that the perimeter of a quadrilateral is larger than the sum of the lengths of its diagonals.
			9.3.3.4	Apply the Pythagorean Theorem and its converse to solve problems and logically justify results. *For example*: When building a wooden frame that is supposed to have a square corner, ensure that the corner is square by measuring lengths near the corner and applying the Pythagorean Theorem.
			9.3.3.5	Know and apply properties of right triangles, including properties of 45-45-90 and 30-60-90 triangles, to solve problems and logically justify results. *For example*: Use 30-60-90 triangles to analyze geometric figures involving equilateral triangles and hexagons. *Another example*: Determine exact values of the trigonometric ratios in these special triangles using relationships among the side lengths.
			9.3.3.6	Know and apply properties of congruent and similar figures to solve problems and logically justify results. *For example*: Analyze lengths and areas in a figure formed by drawing a line segment from one side of a triangle to a second side, parallel to the third side. *Another example*: Determine the height of a pine tree by comparing the length of its shadow to the length of the shadow of a person of known height. *Another example*: When attempting to build two identical 4-sided frames, a person measured the lengths of corresponding sides and found that they matched. Can the person conclude that the shapes of the frames are congruent?
			9.3.3.7	Use properties of polygons—including quadrilaterals and regular polygons—to define them, classify them, solve problems and logically justify results. *For example*: Recognize that a rectangle is a special case of a trapezoid. *Another example*: Give a concise and clear definition of a kite.

	Strand	Standard	No.	Benchmark
9, 10, 11	Geometry & Measurement	Know and apply properties of geometric figures to solve real-world and mathematical problems and to logically justify results in geometry.	9.3.3.8	Know and apply properties of a circle to solve problems and logically justify results. *For example*: Show that opposite angles of a quadrilateral inscribed in a circle are supplementary.
		Solve real-world and mathematical geometric problems using algebraic methods.	9.3.4.1	Understand how the properties of similar right triangles allow the trigonometric ratios to be defined, and determine the sine, cosine and tangent of an acute angle in a right triangle.
			9.3.4.2	Apply the trigonometric ratios sine, cosine and tangent to solve problems, such as determining lengths and areas in right triangles and in figures that can be decomposed into right triangles. Know how to use calculators, tables or other technology to evaluate trigonometric ratios. *For example*: Find the area of a triangle, given the measure of one of its acute angles and the lengths of the two sides that form that angle.
			9.3.4.3	Use calculators, tables or other technologies in connection with the trigonometric ratios to find angle measures in right triangles in various contexts.
			9.3.4.4	Use coordinate geometry to represent and analyze line segments and polygons, including determining lengths, midpoints and slopes of line segments.
			9.3.4.5	Know the equation for the graph of a circle with radius r and center (h,k), $(x-h)^2+(y-k)^2=r^2$, and justify this equation using the Pythagorean Theorem and properties of translations.
			9.3.4.6	Use numeric, graphic and symbolic representations of transformations in two dimensions, such as reflections, translations, scale changes and rotations about the origin by multiples of 90˚, to solve problems involving figures on a coordinate grid. *For example*: If the point (3,-2) is rotated 90˚ counterclockwise about the origin, it becomes the point (2,3).
			9.3.4.7	Use algebra to solve geometric problems unrelated to coordinate geometry, such as solving for an unknown length in a figure involving similar triangles, or using the Pythagorean Theorem to obtain a quadratic equation for a length in a geometric figure.

	Strand	Standard	No.	Benchmark
9, 10, 11	Data Analysis & Probability	Display and analyze data; use various measures associated with data to draw conclusions, identify trends and describe relationships.	9.4.1.1	Describe a data set using data displays, such as box-and-whisker plots; describe and compare data sets using summary statistics, including measures of center, location and spread. Measures of center and location include mean, median, quartile and percentile. Measures of spread include standard deviation, range and inter-quartile range. Know how to use calculators, spreadsheets or other technology to display data and calculate summary statistics.
			9.4.1.2	Analyze the effects on summary statistics of changes in data sets. *For example*: Understand how inserting or deleting a data point may affect the mean and standard deviation. *Another example*: Understand how the median and interquartile range are affected when the entire data set is transformed by adding a constant to each data value or multiplying each data value by a constant.
			9.4.1.3	Use scatterplots to analyze patterns and describe relationships between two variables. Using technology, determine regression lines (line of best fit) and correlation coefficients; use regression lines to make predictions and correlation coefficients to assess the reliability of those predictions.
			9.4.1.4	Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. *For example*: After performing several measurements of some attribute of an irregular physical object, it is appropriate to fit the data to a normal distribution and draw conclusions about measurement error. *Another example*: When data involving two very different populations is combined, the resulting histogram may show two distinct peaks, and fitting the data to a normal distribution is not appropriate.
		Explain the uses of data and statistical thinking to draw inferences, make predictions and justify conclusions.	9.4.2.1	Evaluate reports based on data published in the media by identifying the source of the data, the design of the study, and the way the data are analyzed and displayed. Show how graphs and data can be distorted to support different points of view. Know how to use spreadsheet tables and graphs or graphing technology to recognize and analyze distortions in data displays. *For example*: Shifting data on the vertical axis can make relative changes appear deceptively large.
			9.4.2.2	Identify and explain misleading uses of data; recognize when arguments based on data confuse correlation and causation.
			9.4.2.3	Explain the impact of sampling methods, bias and the phrasing of questions asked during data collection.

	Strand	Standard	No.	Benchmark
9, 10, 11	Data Analysis & Probability	Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.	9.4.3.1	Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities. *For example*: If one girl and one boy are picked at random from a class with 20 girls and 15 boys, there are 20 × 15 = 300 different possibilities, so the probability that a particular girl is chosen together with a particular boy is $\frac{1}{300}$.
			9.4.3.2	Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.
			9.4.3.3	Understand that the Law of Large Numbers expresses a relationship between the probabilities in a probability model and the experimental probabilities found by performing simulations or experiments involving the model.
			9.4.3.4	Use random numbers generated by a calculator or a spreadsheet, or taken from a table, to perform probability simulations and to introduce fairness into decision making. *For example*: If a group of students needs to fairly select one of its members to lead a discussion, they can use a random number to determine the selection.
			9.4.3.5	Apply probability concepts such as intersections, unions and complements of events, and conditional probability and independence, to calculate probabilities and solve problems. *For example*: The probability of tossing at least one head when flipping a fair coin three times can be calculated by looking at the complement of this event (flipping three tails in a row).
			9.4.3.6	Describe the concepts of intersections, unions and complements using Venn diagrams. Understand the relationships between these concepts and the words AND, OR, NOT, as used in computerized searches and spreadsheets.
			9.4.3.7	Understand and use simple probability formulas involving intersections, unions and complements of events. *For example*: If the probability of an event is p, then the probability of the complement of an event is $1 - p$; the probability of the intersection of two independent events is the product of their probabilities. *Another example*: The probability of the union of two events equals the sum of the probabilities of the two individual events minus the probability of the intersection of the events.

	Strand	Standard	No.	Benchmark
			9.4.3.8	Apply probability concepts to real-world situations to make informed decisions. *For example*: Explain why a hockey coach might decide near the end of the game to pull the goalie to add another forward position player if the team is behind. *Another example*: Consider the role that probabilities play in health care decisions, such as deciding between having eye surgery and wearing glasses.
9, 10, 11	Data Analysis & Probability	Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.	9.4.3.9	Use the relationship between conditional probabilities and relative frequencies in contingency tables. *For example*: A table that displays percentages relating gender (male or female) and handedness (right-handed or left-handed) can be used to determine the conditional probability of being left-handed, given that the gender is male.