4. Add parentheses to form the type of compound statement indicated. If no parentheses are needed indicate that fact. (2pts/problem).

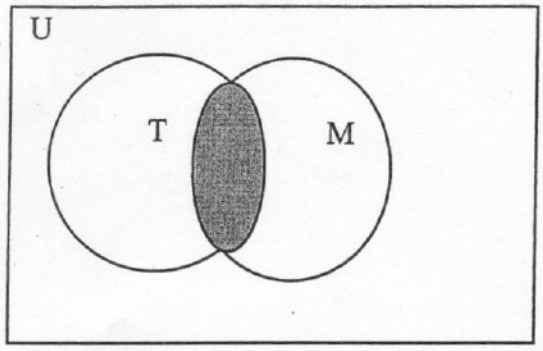
a. Conjunction: $\sim P \wedge (Q \vee R)$

b. Conditional: $\sim R \wedge Q \rightarrow P$ none needed

c. Disjunction: $(P \leftrightarrow \sim Q) \vee R$

5. Write in set notation in **two** different ways. (4 pts/problem).

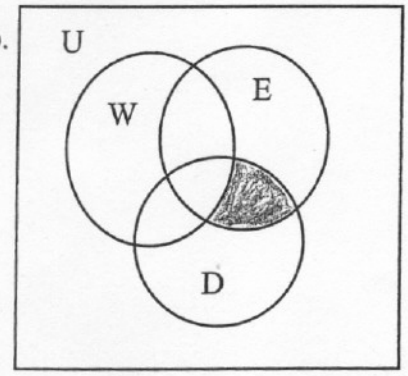
a.



5.a. $T \cap M$ or $(T' \cup M')'$

$(T \cap M)'$ $(T' \cup M')'$

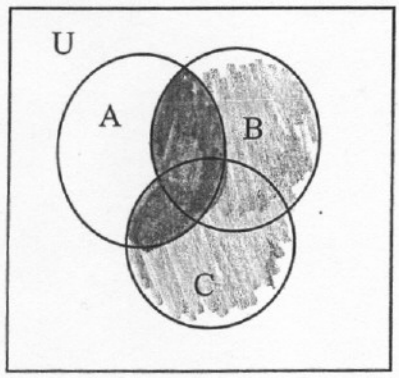
b.

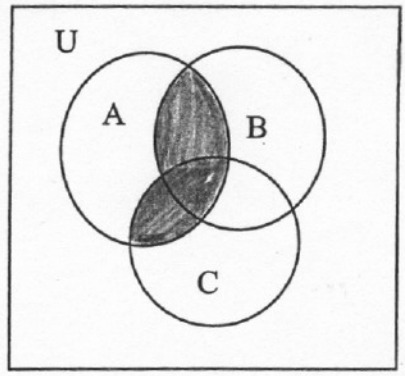


b. $D \cap E - W$ or $D \cap E \cap W'$

$D \cap E \cap W'$

6. a. Use a pair of Venn diagrams to determine whether $A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$. (3pts.)





b. Are they equivalent? yes

+17
17

Key

Name_

Math 110
Test 2

I. Complete as directed showing appropriate work.

1. Identify each sentence as a simple statement, compound statement, or neither. If the statement is compound, then classify it as a negation, conjunction, disjunction, conditional, or biconditional (2 pts/problem).

   a. You can go or I can go. compound, disjunction

   b. When is Halloween? neither

   c. It is not true that you have class Friday and you are finished with logic.

   compound, negation

   d. The car is working. simple

2. By means of the appropriate connectives and parentheses, symbolize each statement, using the given symbols for the simple statements: (2pts/problem)

A = The first frost of the season will occur tonight.

B = It is to get down to 32°.

D = I will cover the plants.

   a. The first frost of the season will occur tonight, but I will cover the plants.

   $A \wedge D$

   b. It is not the case that it is to get down to 32° and the first frost of the season will occur tonight.

   $\sim(B \wedge A)$

   c. The first frost of the season will occur tonight if it is to get down to 32°.

   $B \rightarrow A$

3. How many cases (rows) would be necessary in a truth table if there were four different simple statements p, q, r, s? Show your work for full credit (2 pts).

$2^4 = 16$

16 rows

+16/16

7. Let C = It is nice outside.
R = I will go biking today.
B = I do not have any homework.

Write the following statement $C \wedge \sim B \rightarrow R$ in words. (3 pts)

If it is nice outside and I have homework, then I will go biking today

8. Determine if the argument is valid or invalid via a truth table (3 pts).

$G \wedge H$
$\sim G$

$\sim H$

$(G \wedge H) \wedge \sim G \rightarrow \sim H$

G	H	$(G \wedge H) \wedge \sim G \rightarrow \sim H$				
T	T	T	F	F	T	F
T	F	F	F	F	T	T
F	T	F	F	T	T	F
F	F	F	F	T	T	T
					*	

Valid

9. Given the following argument:

If the Twins win the game, then they will have home field advantage for the pennant.
They did not have home field advantage for the pennant.
Therefore, the Twins did not win the game.

a. Assign statements to letters and then rewrite the argument in symbols **two** different ways. (3 pts)

T = Twins win the game
H = They will have home field advantage for the pennant.

① $T \rightarrow H$
$\sim H$

$\sim T$

② $(T \rightarrow H) \wedge \sim H \rightarrow \sim T$

b. Complete a truth table for the problem (3 pts).

T	H	$(T \rightarrow H) \wedge \sim H \rightarrow \sim T$				
T	T	T	F	F	T	F
T	F	F	F	T	T	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

c. Interpret the results (1 pt).

Valid Argument
Tautology

+ 13
13

10. Construct a truth table for $(P \wedge \sim R) \vee Q$. (4 pts)

P	Q	R	$(P \wedge \sim R) \vee Q$		
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	F
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	F	F

*

11. Determine the truth-value for each of the following. Show your work for full credit. (2 pts/problem)

a. $8(2-4) = 16$ iff $-7-(-15) > 6$

$16 = 16 \longleftrightarrow 8 > 6$

F ⟷ T

11.a. False

b. Fargo is not in Minnesota or Moorhead is not in North Dakota, and London is in France

$(T \quad \vee T) \wedge F \rightarrow T \wedge F$

b. False

c. 5 % of $4 = 0.20$ and $|3-5| > 1$

$2 > 1$

$T \wedge$

c. True

d. If $\sim R \wedge Q$ is true, $R \vee Q$ is True.

e. If $J \rightarrow \sim K$ is false, then J must be True and K must be True.

f. "If you did not get up, then you missed class" is logically equivalent to "Either you did not get up or you missed class."

$\sim G \rightarrow m$

G = you got up

m you missed class

$\sim G \vee m$

f. False

G	M	$\sim G \rightarrow m$		
T	T	F	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

G	m	$\sim G \vee m$		
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	F

12. Write each using set-builder notation (2 pts/problem).

a. { …, -3, -2, -1}

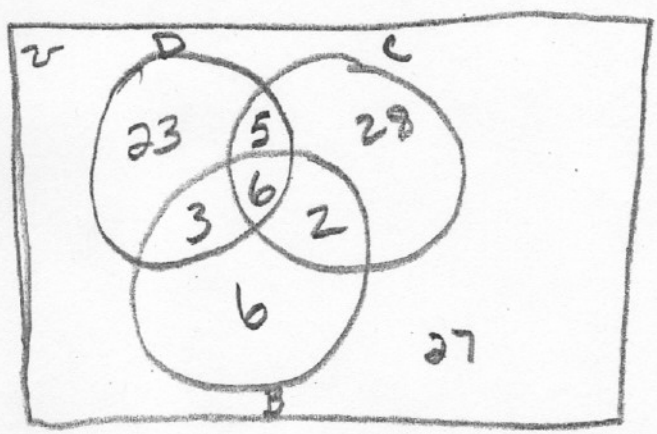
12.a. $\{x \in I \mid x \leq -1\}$

b. {2, 4, 6, 8, …, 50}

b. $\{x \in N \mid x \text{ is even and } x \leq 50\}$

13. In a survey of 100 people it was found that the number have dogs as pets ~~was 37;~~ ~~the number having cats as pets was 41; the number having birds was 17; the number having dogs and cats was 11; the number having cats and birds was 8; the number having dogs and birds was 9; and the number who had dogs, cats, and birds as pets was 6.~~

a. Make and fill in a Venn diagram to illustrate this situation (3 pts).



b. How many do not have any of the three animals? (1 pt) 27

c. How many have at least two of the three animals? (2 pts) 16

14. Complete a truth table for the following $(P \wedge \sim Q) \vee \sim P$ (4 pts)

P	Q	$(P \wedge \sim Q) \vee \sim P$				
T	T	T	F	F	F	F
T	F	T	T	T	T	F
F	T	F	F	F	T	T
F	F	F	F	T	T	T

+14
14

15. Use truth tables to determine if $W \leftrightarrow Z$ is logically equivalent to $(Z \rightarrow W) \vee (W \rightarrow Z)$ (3 pts).

W	Z	W ↔ Z
T	T	T
T	F	F
F	T	F
F	F	T

False not equivalent

W	Z	(Z → W)	∨	(W → Z)
T	T	T	T	T
T	F	T	T	F
F	T	F	T	T
F	F	T	T	T

16. Use De Morgan's law to create equivalent statements for each of the following. (2 pts/prob)

a. $\sim P \wedge Q$

$\sim(\sim P \wedge Q)$

$\sim(P \wedge \sim Q)$

$\sim(P \vee \sim Q)$

$\sim(P \vee \sim Q)$

b. $M \rightarrow \sim N$

$\sim(M \rightarrow \sim N)$ $\sim(M \vee N)$

$\sim(\sim M \vee \sim N)$ $\sim(M \wedge N)$

$\sim(M \wedge N)$

c. You were ill, or you were in class.
(Write using symbols, apply the law, and then translate back to words 3 pts)

I = You were ill

C = you were in class

$I \vee C$

$\sim(I \vee C)$

$\sim(\sim I \vee \sim C)$

$\sim(\sim I \wedge \sim C)$

It is false that you weren't ill and you weren't in class.

d. $(C' \cap D)$ in set theory

$(C' \cap D)'$

$(C \cap D')'$

$(C \cup D')'$

$(C \cup D')'$

+12

12

II. True or False, if false explain why (2 pt/problem).

17. a. A conjunction is true when at least one of the statements is true.

False — a conjunction is true when both statements are T

b. A paradox is neither true nor false.

True — (it contradicts itself)

~~c~~. A tautology occurs when all columns are true.

Drop

d. "It snows only if it is cold" is logically equivalent to "If it snows, then it is cold."

True

e. If A = {4, 6, 7, 8} and B = {5, 6, 7, 9} then the number of one-to-one correspondences between A and B is 16.

False — $4! = 24$ one-to-one correspondences

8/8