Name Key

Math 110
Test 2

I. Complete as directed showing appropriate work.

1. Identify each sentence as a simple statement, compound statement, or neither. If the statement is compound, then classify it as a negation, conjunction, disjunction, conditional, or biconditional (1pt/problem).

   a. Not all spiders are black. Compound - negation

   b. The dog will bite if and only if it is cornered. Compound - biconditional

   c. Spring break is in just 2 ½ weeks away. Simple

   d. Get out of the car. Neither

   e. I decided not to go to class, but I planned to get the notes. Compound - Conjunction

2. By means of the appropriate connectives and parentheses, symbolize each statement, using the given symbols for the simple statements: (2pts/problem)

C = Breakfast includes a cup of coffee
P = A poptart is part of breakfast
J = Juice is part of breakfast

   a. Breakfast includes a cup of coffee, or a poptart and juice.
   $C \vee (P \wedge J)$

   b. Juice is part of breakfast, if a poptart is part of breakfast.
   $P \rightarrow J$

   c. It is not the case that either breakfast includes a cup of coffee or a poptart.
   $\sim (C \vee P)$

   d. Juice is not part of breakfast, but coffee is.
   $\sim J \wedge C$

3. How many cases would be necessary in a truth table if there were five different simple statements p, q, r, s, t ? Show your work for full credit (2 pts).

$2^5 = 32$

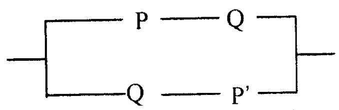
32

4. Add parentheses, if necessary, in each statement to form the type of compound statement indicated (2pts/problem).

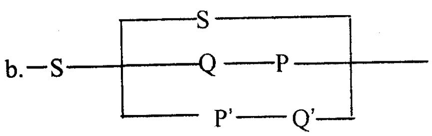
   a. Conjunction: $(P \vee Q) \wedge \sim P$
   b. Negation: $\sim(R \wedge Q \rightarrow P)$
   c. Conditional: $(P \leftrightarrow \sim Q) \rightarrow R$

5. Write a symbolic statement for each network (2pts/problem).

a.



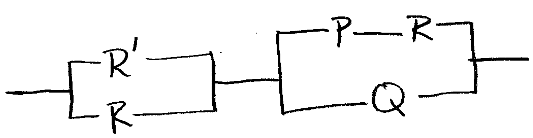
5.a. $(P \wedge Q) \vee (Q \wedge \sim P)$



b. $S \wedge [S \vee (Q \wedge P) \vee (\sim P \wedge \sim Q)]$

or $(S \wedge S) \vee (S \wedge Q \wedge P) \vee (S \wedge \sim P \wedge \sim Q)$

6. Construct a switching network that corresponds to $(\sim R \vee R) \wedge [(P \wedge R) \vee Q]$ (3pts.)



7. Determine if the argument is valid or invalid via a truth table (3 pts).

$G \vee H$
$\sim H$
$\overline{\quad G \quad}$

G	H	$(G \vee H)$	$\wedge$	$\sim H$	$\rightarrow$	G
T	T	T	F	F	T	T
T	F	T	T	T	T	T
F	T	T	F	F	T	F
F	F	F	F	T	T	F
1	2	3	5	4	5/6	1

Valid

8. Let C = It is cold outside.
R = I will go running today.
B = The winter blues are setting in.

Write the following statement $(B \wedge \sim C) \rightarrow R$ in words. (2pts)

If the winter blues are setting in and it is not cold outside, then I will go running today.

9. Given the following argument:

If a student is a freshman, then the student takes English.
The student is a not a freshman.
Therefore, the student does not take English.

a. Assign statements to letters and then rewrite the argument in symbols two different ways. (3 pts)

Let F = a student is a freshman
E = the student takes English

$$\begin{array}{l} F \rightarrow E \\ \sim F \\ \hline \sim E \end{array} \quad \text{or} \quad (F \rightarrow E) \wedge \sim F \rightarrow \sim E$$

b. Complete a truth table for the problem (3 pts).

F	E	$(F \rightarrow E)$	$\wedge$	$\sim F$	$\rightarrow$	$\sim E$
T	T	T	F	F	T	F
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T
1	2	3	5	4	7	6

c. Interpret the results (1 pt).

This is an invalid argument as it is not a tautology

10. Construct a truth table for $\sim (P \wedge R) \vee \sim Q$ (4 pts)

P	Q	R	$\sim$	$(P \wedge R)$	$\vee$	$\sim Q$
T	T	T	F	T	F	F
T	T	F	T	F	T	F
T	F	T	F	T	T	T
T	F	F	T	F	T	T
F	T	T	T	F	T	F
F	T	F	T	F	T	F
F	F	T	T	F	T	T
F	F	F	T	F	T	T
1	2	3	5	4	7	6

11. Determine the truth value for each of the following & show your work: (2 pts/problem)

a. Paris is not in France or Rome is not in Germany, and London is in England.

$(F \vee T) \wedge T$

$T \wedge T$

a. True

b. $9(2 - 4) = 18$ iff $-4 + (+15) < 6$

$9(-2) \neq 18$ $\quad 11 \not< 6$

$F \Leftrightarrow F$

b. True

$T \longrightarrow T$

c. I gave you an assignment implies you have homework

c. True

12. Write each using set-builder notation (2 pts/problem).

a. {3, 4, 5, 6, 7, …}

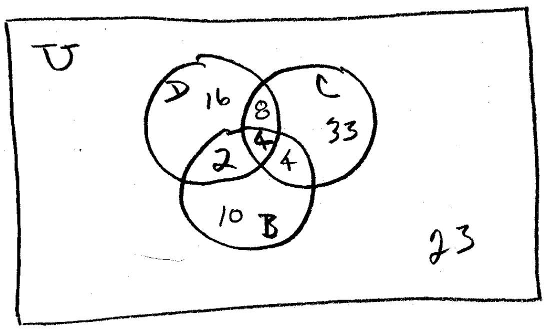
12a. $\{x \in N \mid x > 2\}$

b. { …, -3, -2, -1, 0}

b. $\{x \in J \mid x \leq 0\}$

13. In a survey of 100 people it was found that the number have dogs as pets was 30; the number having cats as pets was 49; the number having birds was 20; the number having dogs and cats was 12; the number having cats and birds was 8; the number having dogs and birds was 6; and the number who had dogs, cats, and birds as pets was 4.

a. Make and fill in a Venn diagram to illustrate this situation (3 pts).



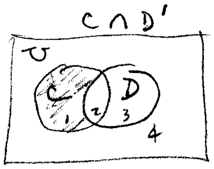
$$\begin{array}{r} 49 \\ 28 \\ \hline 77 \end{array}$$

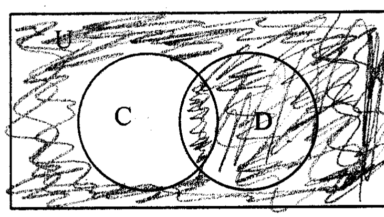
b. How many do not have any of the three animals? (1 pt) 23

c. How many have at least two of the three animals? (2 pts) 18 ← 2+4+8+4

14. Shade the region $(C \cap D')'$ (2 pts)

$C \cap D'$





15. Use De Morgan's laws to create an equivalent expression to $(C \cap D')'$ in set theory. (2 pts)

1) $(C \cap D')'' \equiv (C \cap D')$

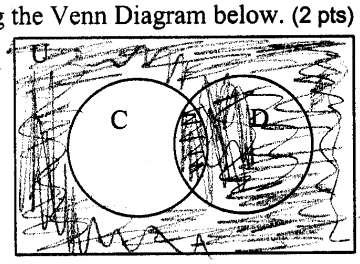
2) $C' \cap D'' \equiv C' \cap D$

3) $C' \cup D$

15. $C' \cup D$

16. Shade your answer to problem number 15 using the Venn Diagram below. (2 pts)
What does it tell you about your solution?

$(C \cap D')' \equiv C' \cup D$



17. Use De Morgan's law to create equivalent statements for each of the following.

a. $P \wedge \sim Q$ (2 pts)

1) $\sim(P \wedge \sim Q)$

2) $\sim(\sim P \wedge \sim\sim Q) \equiv \sim(\sim P \wedge Q)$

3) $\sim(\sim P \vee Q)$

$\sim(\sim P \vee Q)$

b. $\sim(M \rightarrow N)$ (2 pts)

1) $\sim\sim(M \rightarrow N) \equiv M \rightarrow N \equiv \sim M \vee N$

2) $\sim\sim M \vee \sim N \equiv M \vee \sim N$

3) $M \wedge \sim N$

$M \wedge \sim N$

c. You need to work, or you are ill.

(Write using symbols, apply the law, and then translate back to words 3 pts)

$W \vee I$

1) $\sim(W \vee I)$

2) $\sim(\sim W \vee \sim I)$

3) $\sim(\sim W \wedge \sim I)$

It is not the case you don't need to work, and you are not ill.

18. Use truth tables to determine if $W \rightarrow Z$ is logically equivalent to $\sim Z \rightarrow \sim W$ (3 pts).

W	Z	$W \rightarrow Z$
T	T	T
T	F	F
F	T	T
F	F	T

W	Z	$\sim Z$	$\rightarrow$	$\sim W$
T	T	F	T	F
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Yes, they are logically equivalent

19. Symbolize the argument and by means of a truth table, determine whether the argument is valid or invalid (4 pts).

If the machine works, we will be rich. (M, R)
Either the machine works, or we will be rich.
Therefore, we will be rich.

$M \to R$
$M \vee R$
$\overline{\qquad}$
R

Valid

M	R	$(M \to R)$	$\wedge$	$(M \vee R)$	$\Rightarrow$	R
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	F	T	F	F	T	F
1	2	3	5	4	7	2

II. True or False (2 pt/problem).

20. Tell whether each statement is true or false, and for a-e if false explain why

a. A disjunction is true when at least one of the statements is true.

True ______

b. "If you did not get up, then you missed class" is logically equivalent to "Either you did not get up or you missed class."

False If you did not get up, then you missed class ≡ Either you did get up or you missed class
$P \Rightarrow Q \equiv \sim P \vee Q$

c. A tautology means the left and right members of the biconditional are logically equivalent.

True ______

d. The dominant connective in the compound statement "If you go then I will go, or Professor Harms" can go is the or.

True ______

e. If A = {4, 6, 8} and B = { 5, 6, 7, 9} then n(A x B) = 7

False $n(A \times B) = 3 \cdot 4 = 12$

f. If ~R ∧ Q is true, R ∨ ~Q is False.
T T; R = F; F ~T; F ∨ F

g. If ~ J → ~ K is false, then J must be False and K must be True.
T F
J = F & K = T