

**Definition:** A **proposition** is a declarative sentence that is either true or false (but not both). That is, a statement ends up having one of two possible truth values:

- If a statement is true, we assign it the truth value  $T$ . If a statement is false, we assign it the truth value  $F$ . In practice, we will often think of statements as *potentially* having one of the two possible truth values without actually giving it a specific assignment.
- In fact, we may not even give an actual statement. Instead, we use lower case letters such as  $p, q, r, s$  to denote **propositional variables** (unspecified statements).
- We then consider all possible ways of assigning truth values to variables representing statements, often after combining several statements using *logical connectives* to form **compound propositions**.

**Basic Truth Tables:**

The truth table for “not” ( $\neg$ ): Given a simple statement  $p$ . If  $p$  is true, then  $\neg p$  is false. Similarly, if  $p$  is false, then  $\neg p$  is true. Note that the statement  $p$  has only two possible truth values, so there are two rows in the truth table. The following table summarizes this information.

$p$	$\neg p$
$T$	$F$
$F$	$T$

The truth table for “or” ( $\vee$ ): Given two simple statements  $p, q$ , the compound statement  $p \vee q$  has four possible truth value assignments: both could be true, both could be false, the first could be true while the second is false, and the first could be false while the second is true. Since  $\vee$  always represents an *inclusive or*, the statement  $p \vee q$  is true except when  $p$  and  $q$  are both false. The following table summarizes this information.

$p$	$q$	$p \vee q$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$F$

The truth table for “and” ( $\wedge$ ): Given two simple statements  $p, q$ , as above the compound statement  $p \wedge q$  has four truth value assignments. The statement  $p \wedge q$  is only true when  $p$  and  $q$  are both true. The following table summarizes this information.

$p$	$q$	$p \wedge q$
$T$	$T$	$T$
$T$	$F$	$F$
$F$	$T$	$F$
$F$	$F$	$F$

The truth table for a conditional ( $\rightarrow$ ): Given two simple statements  $p, q$ , as above the compound statement  $p \rightarrow q$  has four possible truth value assignments. The statement  $p \rightarrow q$  is true except in the case when  $p$  is true and  $q$  is false. To see this, it is helpful to think about the conditional statement, “If you eat your vegetables then you will get dessert.” When is this a false statement? The following table summarizes this information.

$p$	$q$	$p \rightarrow q$
$T$	$T$	$T$
$T$	$F$	$F$
$F$	$T$	$T$
$F$	$F$	$T$

The truth table for a biconditional ( $\leftrightarrow$ ): Given two simple statements  $p, q$ , as above the compound statement  $p \leftrightarrow q$  has four possible truth value assignments. The statement  $p \leftrightarrow q$  is true when  $p$  and  $q$  are both true and when  $p$  and  $q$  are both false. When  $p$  and  $q$  have opposite truth values, the statement  $p \leftrightarrow q$  is false. The following table summarizes this information.

$p$	$q$	$p \leftrightarrow q$
$T$	$T$	$T$
$T$	$F$	$F$
$F$	$T$	$F$
$F$	$F$	$T$