





3.1.2 Historical Overview of Transformational Geometry

I begin to understand that while logic is a most excellent guide to governing our reason, it does not, as regards stimulation to discovery, compare with the power of sharp distinction which belongs to geometry.

—  [Galileo Galilei \(1564–1643\)](#)


During the 17th century,  [René Descartes](#) (1596–1650), a French mathematician and philosopher, laid the stepping stones for modern mathematics, specifically in analytical geometry, through the use of the Cartesian coordinate system. That is, every point of a curve is given two numbers that represent its location in a plane. Descartes' system was more general than the usage of horizontal and vertical axes as we know it today. This new coordinate system helped to show that there was a link between geometry and algebra, starting with a geometric shape or curve and assigning an ordered pair to each point so algebraic techniques could be assigned to the figure.

 [Pierre de Fermat](#) (1601–1665), a French jurist, proposed a system of analytical geometry similar to the one noted by Descartes. Fermat's system used a more direct approach and is more similar to the system currently used. Fermat and Descartes are both credited with independently developing the ideas of analytical geometry. Fermat is credited with contributions to every field of mathematics known in the 17th century.


This idea of assigning algebraic ideas to geometric figures led to the study of group theory in geometry.  [Felix Klein](#) (1849–1925), a German geometer, showed the importance of groups in geometry. This new idea allowed Klein to unify geometry. Klein's 1872 address in Erlanger, Germany proposed that the study of geometry be defined as the study of transformations that leave objects invariant (unchanged). This viewpoint is now known as the  [Erlanger Program](#). Klein's address was able to rearrange what seemed to be unrelated geometries known at this time into a cohesive system. His lecture classified geometry to include both Euclidean and non-Euclidean geometries.







Transformational and analytical geometry are not new branches of geometry; they are considered to be part of Euclidean geometry. These approaches include new methods of working out geometric questions. The geometric designs involved used many ideas that have long been a part of art and culture.


Different transformations have been used in art, architecture, crafts, and quilts throughout history. Historians have found numerous transformation designs in pottery, architecture, rugs, quilts, and art pieces from almost every known culture. The designs used can help to determine where and to whom an artifact belonged.




There are many reasons why historians believe that cultures started to use transformations in their art work. For example,  [Arabesque](#) developed in the 7th, 8th, and 9th centuries from beliefs that creating living objects in art was blasphemous or that only God should create animals and other living objects. Due to this belief, many did not use living creatures in art work, instead they used different transformations and geometric designs to increase the appeal of their art and architecture.


Another example why historians believe transformations were used was a lack of materials, such as in the history of quilting. When there was a shortage of quilting materials, women would often turn to a

technique called appliqué to visually increase the appeal of their work.  [Appliqué](#), a process when one piece of fabric is sewn onto another and then stitched together with an intricate design, traditionally had elaborate geometric transformations that were typically symmetric; however, modern appliqué has been expanded to include pictures of everyday items, such as flowers and houses that are not necessarily symmetric.

Another reason historians believe cultures would use transformations was the aesthetic appeal that geometric designs create. Different cultures loved the visual interest and equality that geometric transformations made in the art and architecture of their communities ( [Islamic Art](#)). These techniques were typically inexpensive to add to crafts, such as pottery. They also liked the aesthetic appeal of symmetric buildings. This idea can be seen in buildings throughout history, from the  [Parthenon](#) of ancient Greece to  [St. Peter's Basilica](#) in Rome from the 17th century to  [St. Peter's Cathedral](#) in Adelaide in the 19th century, as well as over many centuries in  [Islamic architecture](#). Even modern buildings from the 19th and 20th Centuries are built on the basis of symmetry such as the  [United States Capitol](#) building in Washington, D.C.



 [Transformational geometry](#) is quite important in many fields, such as the study of architecture, anthropology, and art, to name a few. The study of which forms of transformations were used helps to distinguish time frames for artifacts and helps to illustrate which cultures may have made the item being studied.

For example, architects are able to study the history of very old buildings, taking note of which transformations were used. A classical example that involves this study is the illustration of the study of the history of the  [Parthenon](#) in Athens, Greece. This was a shrine constructed in honor of the goddess  [Athena](#), built between 447 and 436 B.C. Historians look at the different  [frieze patterns](#) throughout the temple to help understand the culture and architecture. They are able to compare the designs from the Parthenon to other buildings throughout Greece and Europe to help see which cultures may have had contact with the Athenians. They can also compare the frieze patterns used in other buildings to see how similar they are to the designs in the Parthenon and are able to help narrow in on a time frame for when the other building was constructed.

Historians can also use different transformations to help learn for what a building may have been used. Historically, buildings which were very well constructed and decorated typically served religious purposes, honored a god or king, or served a wealthy or important person in the community. If a building has lavish frieze patterns or other transformations, historians are more thoroughly able to recognize how the building was used and understand the importance of the building in the community studied. A familiar instance of this throughout history is the importance of churches and palaces to a particular culture. For example the  [Alhambra Palace](#), built in Granada, Spain in the 13th or 14th century, has many extravagant symmetric and tessellation designs throughout the tile work, windows, and ceilings in the building. This lavish detail throughout historic architecture is usually because of the importance of the building for the community. The Alhambra Palace, which includes many intricate designs throughout its construction, helps to show how important the building was to the community and dynasty as well as the wealth and importance of the population.

Transformations are also studied heavily by anthropologists. Many cultures throughout time have used transformations in their craft and pottery work. Anthropologists are able to study the artifacts of cultures through pottery; pottery is a common remain due to the durability of materials used. Anthropologists are able to study these design features to help understand which cultures had contact with each other and present evidence of their lives. Pottery artifacts are studied a great deal to help

understand the culture. One way they are able to do that is by the transformations that were used. Cultures typically used the same designs in most of their work, so anthropologists are able to match a design with a particular culture. If they find the same design in an artifact from the site of a different culture, they are able to establish that the cultures had contact either through trade or education. This is important for anthropologists to understand how a culture lived and worked and to track the interactions and movements between several cultures.

Another area that transformational geometry is commonly used is in art. There have been many artists throughout history that have used different techniques of transformations, such as symmetry and tessellations, in their pieces. A well known artist using transformations was  [M. C. Escher](#) (1898–1972), a Dutch graphic artist. He would typically use symmetry and tessellations in his art. The Alhambra Palace, previously mentioned for its lavish transformational detail, was a common place that Escher would go to work on his pieces. Escher read a few mathematics papers regarding symmetry, specifically  [George Pólya](#)'s (1887–1985) 1924 paper on 17 plane symmetry groups, and although he did not understand many of the ideas and the mathematical theory of why it worked, he did understand the concepts of the paper and was able to apply the ideas in his work. These concepts helped him to use mathematics more extensively throughout many of his later pieces.

Transformational geometry has been extensively used throughout history by cultures from all regions to create works of art and decorate their architectural buildings. This use of transformations, no matter the reason, is very helpful for historians and anthropologists in aiding with the research of different cultures and their traditions.

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[Timothy Peil](#)

[Mathematics Dept.](#)

[MSU Moorhead](#)

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