STEP 1 State the hypotheses, and select α. For factor A, the null hypothesis states that there is no difference in the amount eaten for non-obese versus obese participants. In symbols,

$$H_0: \quad \mu_{A_1} = \mu_{A_2}$$

$$H_1: \quad \mu_{A_1} \neq \mu_{A_2}$$

For factor B, the null hypothesis states that the amount eaten will be the same for full-stomach participants as for empty-stomach participants. In symbols,

$$H_0: \quad \mu_{B_1} = \mu_{B_2}$$

$$H_1: \quad \mu_{B_1} \neq \mu_{B_2}$$

For the $A \times B$ interaction, the null hypothesis can be stated two different ways. First, the difference in eating between the full-stomach and empty-stomach conditions will be the same for non-obese and obese participants. Second, the difference in eating between the non-obese and obese participants will be the same for the full-stomach and empty-stomach conditions. In more general terms,

H_0: The effect of factor A does not depend on the levels of factor B (and B does not depend on A).

H_1: The effect of one factor does depend on the levels of the other factor.

We will use $\alpha = .05$ for all tests.

STEP 2 Locate the critical region. To locate the critical values for each of the three F-ratios, we first must determine the df values. For these data (Table 15.6),

Notice that the F distribution table has no entry for $df = 1, 76$. A close and conservative estimate of this critical value may be obtained by using $df = 1, 70$ (critical $F = 3.98$). Whenever there is no entry for the df value of the error term, use the nearest smaller df value in the table.

$$df_{\text{total}} = N - 1 = 79$$

$$df_{\text{between treatments}} = (\text{number of cells}) - 1 = 3$$

$$df_{\text{within treatments}} = \Sigma(n - 1) = 76$$

$$df_A = (\text{number of levels of } A) - 1 = 1$$

$$df_B = (\text{number of levels of } B) - 1 = 1$$

$$df_{A\times B} = df_{\text{between treatments}} - df_A - df_B = 1$$

Thus, all three F-ratios will have $df = 1, 76$. With $\alpha = .05$, the critical F value is 3.98 for all three tests.

STEP 3 Use the data to compute the F-ratios. First, we will analyze the SS values:

$$SS_{\text{total}} = \Sigma X^2 - \frac{G^2}{N} = 31{,}836 - \frac{1440^2}{80}$$

$$= 31{,}836 - 25{,}920$$

$$= 5916$$

$$SS_{\text{between treatments}} = \Sigma\frac{T^2}{n} - \frac{G^2}{N}$$

$$= \frac{440^2}{20} + \frac{300^2}{20} + \frac{340^2}{20} + \frac{360^2}{20} - \frac{1440^2}{80}$$

$$= 26{,}440 - 25{,}920$$

$$= 520$$

$$SS_{\text{within treatments}} = \Sigma SS_{\text{inside each cell}}$$

$$= 1540 + 1270 + 1320 + 1266$$

$$= 5396$$

For factor A (weight), the two row totals are $T_{\text{non-obese}} = 740$ and $T_{\text{obese}} = 700$. Each row total is obtained by summing 40 scores. Using these values, the SS for factor A is

$$SS_{\text{factor } A} = \Sigma\frac{T^2_{\text{ROW}}}{n_{\text{ROW}}} - \frac{G^2}{N}$$

$$= \frac{740^2}{40} + \frac{700^2}{40} - \frac{1440^2}{80}$$

$$= 25{,}940 - 25{,}920$$

$$= 20$$

For factor B (fullness), the two column totals are $T_{\text{empty}} = 780$ and $T_{\text{full}} = 660$. Each of these column totals is the sum of 40 scores. Using these values, the SS for factor B is

$$SS_{\text{factor } B} = \Sigma \frac{T^2_{\text{COLUMN}}}{n_{\text{COLUMN}}} - \frac{G^2}{N}$$

$$= \frac{780^2}{40} + \frac{660^2}{40} - \frac{1440^2}{80}$$

$$= 26{,}100 - 25{,}920$$

$$= 180$$

$$SS_{A\times B} = SS_{\text{between treatments}} - SS_A - SS_B$$

$$= 520 - 20 - 180$$

$$= 320$$

The MS values needed for the F-ratios are

$$MS_A = \frac{SS_A}{df_A} = \frac{20}{1} = 20$$

$$MS_B = \frac{SS_B}{df_B} = \frac{180}{1} = 180$$

$$MS_{A\times B} = \frac{SS_{A\times B}}{df_{A\times B}} = \frac{320}{1} = 320$$

$$MS_{\text{within treatments}} = \frac{SS_{\text{within treatments}}}{df_{\text{within treatments}}} = \frac{5396}{76} = 71$$

Finally, the F-ratios are

$$F_A = \frac{MS_A}{MS_{\text{within treatments}}} = \frac{20}{71} = 0.28$$

$$F_B = \frac{MS_B}{MS_{\text{within treatments}}} = \frac{180}{71} = 2.54$$

$$F_{A\times B} = \frac{MS_{A\times B}}{MS_{\text{within treatments}}} = \frac{320}{71} = 4.51$$

STEP 4 Make decisions. For these data, factor A (weight) has no significant effect; $F(1, 76) = 0.28$. Statistically, there is no difference in the number of crackers eaten by non-obese versus obese participants.

Similarly, factor B (fullness) has no significant effect; $F(1, 76) = 2.54$. Statistically, the number of crackers eaten by full participants is no different from the number eaten by hungry participants. (*Note:* This conclusion concerns the combined group of non-obese and obese participants. The interaction concerns these two groups separately. See page 500 for information concerning the interpretation of results when there is a significant interaction.)

These data produce a significant interaction; $F(1, 76) = 4.51$, $p < .05$. This means that the effect of fullness does depend on weight. Specifically, the degree of fullness did affect the non-obese participants, but it had no effect on the obese participants.

As we saw in Chapter 13, the results from an ANOVA can be organized in a summary table, which shows all the components of the analysis (*SS*, *df*, etc.) as well as the final *F*-ratios. For this example, the summary table would be as follows:

SOURCE	*SS*	*df*	*MS*	*F*
Between treatments	520	3		
Factor *A* (weight)	20	1	20	0.28
Factor *B* (fullness)	180	1	180	2.54
$A \times B$ interaction	320	1	320	4.51
Within treatments	5396	76	71	
Total	5916	79		

MEASURING EFFECT SIZE FOR THE TWO-FACTOR ANOVA

The general technique for measuring effect size with an analysis of variance is to compute a value for η^2, the percentage of variance that is explained by the treatment effects. For a two-factor ANOVA, we will compute three separate values for eta squared: one measuring how much of the variance is explained by the main effect for factor *A*, one for factor *B*, and a third for the interaction. As we did with the repeated-measures ANOVA (page 460) we will remove any variability that can be explained by other sources before we calculate the percentage for each of the three specific effects. Thus, for example, before we compute the η^2 for factor *A*, we will remove the variability that is explained by factor *B* and the variability explained by the interaction. The resulting equation is,

$$\text{for factor } A,\ \eta^2 = \frac{SS_A}{SS_{\text{total}} - SS_B - SS_{A\times B}} \tag{15.11}$$

Note that the denominator of Equation 15.11 consists of the variability that is explained by factor *A* and the other *unexplained* variability. Thus, an equivalent version of the equation is,

$$\text{for factor } A,\ \eta^2 = \frac{SS_A}{SS_A + SS_{\text{within treatments}}} \tag{15.12}$$

Similarly, the η^2 formulas for factor *B* and for the interaction are as follows:

$$\text{for factor } B,\ \eta^2 = \frac{SS_B}{SS_{\text{total}} - SS_A - SS_{A\times B}} = \frac{SS_B}{SS_B + SS_{\text{within treatments}}} \tag{15.13}$$

$$\text{for } A \times B,\ \eta^2 = \frac{SS_{A\times B}}{SS_{\text{total}} - SS_A - SS_B} = \frac{SS_{A\times B}}{SS_{A\times B} + SS_{\text{within treatments}}} \tag{15.14}$$

Because each of the η^2 equations computes a percentage that is not based on the total variability of the scores, the results are often called *partial* eta squares. For the data in Example 15.2, the equations produce the following values:

$$\eta^2 \text{ for factor } A \text{ (weight)} = \frac{20}{5916 - 180 - 320} = \frac{20}{5416} = 0.004 \quad \text{(or 0.4\%)}$$

$$\eta^2 \text{ for factor } B \text{ (fullness)} = \frac{180}{5916 - 20 - 320} = \frac{180}{5576} = 0.032 \quad \text{(or 3.2\%)}$$

$$\eta^2 \text{ for } A \times B = \frac{320}{5916 - 20 - 180} = \frac{320}{5716} = 0.056 \quad \text{(or 5.6\%)}$$

IN THE LITERATURE

REPORTING THE RESULTS OF A TWO-FACTOR ANOVA

The APA format for reporting the results of a two-factor analysis of variance follows the same basic guidelines as the single-factor report. First, the means and standard deviations are reported. Because a two-factor design typically involves several treatment conditions, these descriptive statistics usually are presented in a table or a graph. Next, the results of all three hypothesis tests (*F*-ratios) are reported. For the research study in Example 15.2, the report would have the following form:

> The means and standard deviations are presented in Table 1. The two-factor analysis of variance showed no significant main effect for the weight factor, $F(1, 76) = 0.28, p > .05, \eta^2 = 0.004$; and no significant main effect for the fullness factor, $F(2, 76) = 2.54, p > .05, \eta^2 = 0.032$; but the interaction between weight and fullness was significant, $F(1, 76) = 4.51, p < .05, \eta^2 = 0.056$.
>
> **TABLE 1**
>
> Mean number of crackers eaten in each treatment condition

		FULLNESS	
		EMPTY STOMACH	FULL STOMACH
WEIGHT	NON-OBESE	$M = 22.0$ $SD = 9.00$	$M = 15.0$ $SD = 8.18$
	OBESE	$M = 17.0$ $SD = 8.34$	$M = 18.0$ $SD = 8.16$

You should recognize the elements of this report as being identical to other examples reporting the results from an analysis of variance. Each of the obtained *F*-ratios is reported with its *df* values in parentheses. The term *significant* is used to indicate that the null hypothesis was rejected (the mean differences are greater than what would be expected by chance), and the term *not significant* indicates that the test failed to reject the null hypothesis. The *p* value reported with each *F*-ratio reflects the alpha level used for the test. For example, $p < .05$ indicates that the probability is less than .05 that the obtained mean difference is simply due to chance or sampling error. When an exact value of *p* is available, usually from a computer program, it should be reported instead of using $p < \alpha$. Finally, the measure of effect size is reported for each test. ❑

INTERPRETING INTERACTIONS AND MAIN EFFECTS: SIMPLE MAIN EFFECTS

When a two-factor ANOVA produces a significant interaction, you should be very cautious about accepting the main effects (whether significant or not significant) at face value. In particular, a significant interaction can distort, conceal, or exaggerate the main effects. Therefore, the best advice is to start with the interaction, not the main effects, as the basis for interpreting the results of the study.