Solutions: Chapter 4 Problems

3. set up an index, i.

$N := 8 \qquad i := 1, 2 .. N$

$x_i :=$

1.52660
1.52974
1.52592
1.52731
1.52894
1.52804
1.52685
1.52793

$xbar := \frac{\sum_i x_i}{N} \qquad xbar = 1.52767$ average 1.52767

$s := \sqrt{\frac{\sum_i (x_i - xbar)^2}{N - 1}} \qquad s = 1.25991 \times 10^{-3}$ sample std. deviation. = 0.00126

$var := s^2 \qquad var = 1.5873696 \times 10^{-6}$ $var = 1.59 \times 10^{-6}$

9. Confidence interval is a range of values about the measured/determined mean value, within which the true value of the measured quantity is likely to be.

12. $N := 6 \qquad i := 1, 2 .. N$ ±

$x_i :=$

0.13
0.12
0.16
0.17
0.20
0.11

$xbar := \frac{\sum_{i=1}^{N} x_i}{N} \qquad xbar = 0.14833$ = 0.15 two decimal places

$s := \sqrt{\frac{\sum_{i=1}^{N} (x_i - xbar)^2}{N - 1}} \qquad s = 0.0343$ = 0.03

from tables for 5 degrees of freedom (i.e. N-1), at 90% confidence:

$t := 2.015$

confidence interval = $\frac{t \cdot s}{\sqrt{N}} = 0.02822$ = 0.03

confidence limits are 0.15 ± 0.03

from tables for 5 degrees of freedom (i.e. N-1), at 99% confidence:

$t := 4.032$

confidence interval = $\frac{t \cdot s}{\sqrt{N}} = 0.05646$ = 0.06

confidence limits are 0.15 ± 0.06

13. $xbar := 1.52793$

$N := 7$

$s := 0.00007$

from tables for 6 degrees of freedom (i.e. N-1), at 99% confidence:

$t := 3.707$

confidence interval = $\frac{t \cdot s}{\sqrt{N}} = 9.8078 \times 10^{-5}$ note: t carries 3 decimal plaes

confidence limits are 1.527 93 ± 0.000 10

15. $N := 6$ $i := 1, 2 .. N$

$x1_i :=$ $x2_i :=$

Let x1 and x2 show data from methods 1 and 2 respectively.

x1	x2
0.88	0.83
1.15	1.04
1.22	1.39
0.93	0.91
1.17	1.08
1.51	1.31

$d_i := x1_i - x2_i$ differences for a given sample i from the two methods.

$$dbar := \frac{\sum_{i=1}^{N} d_i}{N}$$

$dbar = 0.05$ = -0.00070

standard deviation of the differences:

$$s_d := \sqrt{\frac{\sum_{i=1}^{N} \left(d_i - dbar\right)^2}{N-1}}$$

$s_d = 0.1241$

Number of degrees of freedom: $N - 1 = 5$

calculation of students t

$$t_{calc} := \frac{dbar}{s_d} \cdot \sqrt{N}$$

$\left|t_{calc}\right| = 0.98693$ = 0.98 < 2.571, students t for 5 degrees of freedom at 95% CL

Difference NOT significant.

20. $N1 := 28$ $N2 := 18$ $N3 := 29$ N1, N2, N3 refers to data from indicator 1, 2 and 3 respectively.

$s1 := 0.00225$ $s2 := 0.00098$ $s3 := 0.00113$

$xbar1 := 0.09565$ $xbar2 := 0.08686$ $xbar3 := 0.08641$

For the first and second data sets

$$s_{pooled} := \sqrt{\frac{(N1-1) \cdot s1^2 + (N2-1) \cdot s2^2}{N1 + N2 - 2}}$$

$s_{pooled} = 1.86483 \times 10^{-3}$

$$t_{calc} := \frac{\left|xbar1 - xbar2\right|}{s_{pooled}} \cdot \sqrt{\frac{N1 \cdot N2}{N1 + N2}}$$

$t_{calc} = 15.60219$ obviously much greater than the t for 44 degrees of freedom (~2.02). So the difference is significant.

For the third and second data sets

$$s_{pooled} := \sqrt{\frac{(N3-1)\cdot s3^2 + (N2-1)\cdot s2^2}{N3+N2-2}}$$

$$s_{pooled} = 1.07579 \times 10^{-3}$$

$$t_{calc} := \frac{|xbar3 - xbar2|}{s_{pooled}} \cdot \sqrt{\frac{N3\cdot N2}{N3+N2}}$$

$t_{calc} = 1.39402$ 1.39 < 2.02 Difference not significant.

24. $x_{large} := 216$ $x_{small} := 192$ $x_q := 216$ $x_n := 204$

$$Q := \frac{|x_q - x_n|}{x_{large} - x_{small}}$$

subscript q and n refer to questionable data point and the data point nearest to it.

$Q = 0.5$ 0.5 < 0.64 from the table at 90% confidence for N=5
KEEP.

26. This question deals with expressing numbers to the correct significant figure. Recognize that b is the value of y when x =0. Simply put b is a y value.

m = -1.298 72 x 10^4 s_m = 13.910

Expressing in the scientific notation with the same exponent: s_m = 0.0013910 x 10^4

Rewriting: m = -1.298 72 x 10^4 ± 0.0013910 x 10^4 : m is significant upto the fourth digit, note 72 is written with a break. Regardless the first non zero position in the error dictates the decimal point in the final answer.

m = -1.299 x 10^4 ± 0.001 x 10^4 answer.

The decimal place in the uncertainty dictates that of of b in this case.

b = 256.695 ± 392.9 = 256.7 ± 392.9