This chapter presents a qualitative treatment of molecular orbitals for chemical bonding (covalent bonds) using the simplest molecule,  $H_2^+$ .

$$H + H^+ \longrightarrow H_2^+$$

 $H_2^{\,*}$  is more stable than widely separated H atom and an H^ ion because of the delocalization of the electron over the molecule and localization of the electron in the region between the two nuclei.

As the participant atomic species come together to form a bond the energy of the molecule move through a minimum energy position (equilibrium bond distance = bond length).

# Chemical Bond in Diatomic Molecules Chapter 23







Covalent bond resembles the superposition of charge distribution (valence electrons) among individual atoms - delocalization. Core electrons remain localized on individual atoms.

Molecular wave function;

$$\psi_i^{molecule} = \psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n, \mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_m)$$



Molecular wave function  $\psi_{i}^{molecule} = \psi(\mathbf{r}_{1}, \mathbf{r}_{2}, ..., \mathbf{r}_{n}, \mathbf{R}_{1}, \mathbf{R}_{2}, ..., \mathbf{R}_{n})$ electron

#### The Born-Oppenheimer approximation

The nuclei have much larger masses in comparison to electron mass and will have a negligibly small velocity. Born-Oppenheimer takes advantage of this fact and makes the assumption that since the nuclei are much heavier in mass compared to the electron, their motion can be ignored while solving the electronic Schrödinger equation.

That is, the nuclei are assumed to be stationary while electrons move around it.

Therefore the motion of the nuclei and the electrons can be separated. The nuclear-nuclear motion/repulsion issues can be treated independently.

The Born-Oppenheimer approximation simplifies the handling of the Schrödinger equation for a molecule.



mwiki.ucdavis.edu/Theoretical\_Chemistry/Chemical\_Bonding/General\_Principles/Born\_Oppenheimer\_Approx

Solve electron motion for a given set of R<sub>i</sub>s.



## Molecular Orbitals

Once electronic energy  $E_{el}$  and wave functions (molecular orbitals) are solved with SE for a given nuclear configuration  $R_{AB}$ , the nuclear repulsion energy is then added for a given nuclear configuration  $R_{AB}$  on top of  $E_{el}$  to obtain the total energy at  $R_{AB}$ .

Change  $\mathsf{R}_{\mathsf{AB}}$  and repeat the process.



# 

# LCAO-MO Model - H<sub>2</sub>

The spaces in which electrons can reside in a molecule (MO) extends over the molecule. Molecular orbital description of chemical bond is based on the principle that;  $\psi_{el} = \psi$  can be written as a linear combination of atomic orbitals on individual atoms LCAO in the molecule.

For the simplest molecule, the simplest MO is constructed by the LC of the AOs  $\phi_a$  and  $\phi_b$  (basis functions) coming from atoms H<sub>a</sub> and H<sub>b</sub>.



 $\psi_1 = c_a \phi_a + c_b \phi_b$ 

an approximation

# $\psi_1 = c_a \phi_a + c_b \phi_b$

The next step is to solve for coefficients  $c_i$  by the variation method, minimize  $<\epsilon>$  value.

Expression for  $<\epsilon>$ ;

$$\langle \varepsilon \rangle = \frac{\int \psi_1^* \hat{H}_{el} \psi_1 d\tau}{\int \psi_1^* \psi_1 d\tau}$$

$$\langle \varepsilon \rangle = \frac{(c_a)^2 \int \phi_a^* \hat{H}_{el} \phi_a \, d\tau + (c_b)^2 \int \phi_b^* \hat{H}_{el} \phi_b \, d\tau + 2c_a c_b \int \phi_a^* \hat{H}_{el} \phi_b \, d\tau}{(c_a)^2 \int \phi_a^* \phi_a \, d\tau + (c_b)^2 \int \phi_b^* \phi_b \, d\tau + 2c_a c_b \int \phi_a^* \phi_b \, d\tau}$$

Assumptions:

$$\int \phi_a^* \widehat{H} \phi_b d\tau = \int \phi_b^* \widehat{H} \phi_a d\tau \quad \text{and} \quad \int \phi_a^* \phi_b d\tau = \int \phi_b^* \phi_a d\tau$$

Bra-ket notation

 $\left\langle \boldsymbol{\phi}_{a}^{*} \mid \widehat{\boldsymbol{H}} \mid \boldsymbol{\phi}_{b} \right\rangle \!=\! \left\langle \boldsymbol{\phi}_{b}^{*} \mid \widehat{\boldsymbol{H}} \mid \boldsymbol{\phi}_{a} \right\rangle \quad \text{and} \quad \left\langle \boldsymbol{\phi}_{a}^{*} \mid \boldsymbol{\phi}_{b} \right\rangle \!=\! \left\langle \boldsymbol{\phi}_{b}^{*} \mid \boldsymbol{\phi}_{a} \right\rangle$ 

Assumption valid for homonuclear di-atomics.

$$\langle \varepsilon \rangle = \frac{(c_a)^2 \int \phi_a^* \hat{H}_{el} \phi_a \, d\tau + (c_b)^2 \int \phi_b^* \hat{H}_{el} \phi_b \, d\tau + 2c_a c_b \int \phi_a^* \hat{H}_{el} \phi_b \, d\tau}{(c_a)^2 \int \phi_a^* \phi_a \, d\tau + (c_b)^2 \int \phi_b^* \phi_b \, d\tau + 2c_a c_b \int \phi_a^* \phi_b \, d\tau}$$

Defining: 
$$\int \phi_a^* \widehat{H} \phi_a d\tau = H_{aa} < 0$$
$$\int \phi_a^* \widehat{H} \phi_b d\tau = H_{ab} < 0$$
$$\int \phi_a^* \phi_a d\tau = S_{aa} = 1$$
$$\int \phi_a^* \phi_b d\tau = S_{ab}; \quad 0 < S_{ab} < 1$$

Rewriting:  $\langle \varepsilon \rangle = \frac{(c_a)^2 H_{aa} + (c_b)^2 H_{bb} + 2c_a c_b H_{ab}}{(c_a)^2 + (c_b)^2 + 2c_a c_b S_{ab}}$ 

 $\int \phi_a^* \widehat{H} \phi_a d\tau = H_{aa} \text{ interaction energy electron } \boldsymbol{a} \text{ to nucleus } \boldsymbol{a}$  $\int \phi_a^* \widehat{H} \phi_b d\tau = H_{ab} \text{ interaction energy nucleus } \boldsymbol{a} \text{ to nucleus } \boldsymbol{b}$  $\int \phi_a^* \phi_a d\tau = S_{aa} = 1 \text{ overlap of a. o. of atom A}$  $\int \phi_a^* \phi_b d\tau = S_{ab} \text{ overlap of a. o. of A and a. o. of B}$ 

Exchange integral;  $H_{ab} < 0$ 

Overlap integral;  $0 < S_{ab} < 1$ 

 $(2c_b + 2c_aS_{ab})\varepsilon$ 

$$H_{ab} = \int \phi_a^* \widehat{H} \phi_b d\tau \le 0$$
  
$$\swarrow S_{ab} = \int \phi_a^* \phi_b d\tau > 0$$

N. B.

. . . . . .

not in individual atoms!!



The larger the overlap (wavefunctions in phase) lower the energy of the system  $<\epsilon>$ , more stable is the covalent bond.

Determining the lowest energy use the variation method.

Differentiation of  $<\varepsilon>$  expression w.r.t the  $c_i$  parameters and equating to zero (to find coefficients);

$$(2c_a + 2c_b S_{ab})\varepsilon = 2c_a H_{aa} + 2c_b H_{ab}$$

$$= 2c_bH_{bb} + 2c_aH_{ab}$$

$$c_a(H_{aa} - \varepsilon) + c_b(H_{ab} - \varepsilon S_{ab}) = 0$$
  
 $c_a(H_{ab} - \varepsilon S_{ab}) + c_b(H_{bb} - \varepsilon) = 0$   
Secular equations

For non-trivial solution the determinant;

$$\begin{vmatrix} H_{aa} - \varepsilon & H_{ab} - \varepsilon S_{ab} \\ H_{ab} - \varepsilon S_{ab} & H_{bb} - \varepsilon \end{vmatrix} = 0 \quad \text{Secular determinant}$$

The order of the determinant depends on (equal to) the # AO s of the basis set used.

Expand the determinant:

Yields a polynomial in  $\varepsilon$ , solution of which will give the energies of the system.

$$\begin{split} \varepsilon &= \frac{1}{2 - 2S_{ab}^2} [H_{aa} + H_{bb} - 2S_{ab}H_{ab}] \pm \frac{1}{2 - 2S_{ab}^2} \\ &\times \left[ \sqrt{\left(H_{aa}^2 + 4H_{ab}^2 + H_{bb}^2 - 4S_{ab}H_{ab}H_{bb} - 2H_{aa}(H_{bb} + 2S_{ab}H_{ab} - 2S_{ab}^2H_{bb})} \right) \right] \end{split}$$

For the homonuclear diatomic,

$$\varepsilon_1 = \frac{H_{aa} + H_{ab}}{1 + S_{ab}}$$
 and  $\varepsilon_2 = \frac{H_{aa} - H_{ab}}{1 - S_{ab}}$ 

vergy

 $\begin{vmatrix} H_{aa} - \varepsilon & H_{ab} - S_{ab}\varepsilon \\ H_{ba} - S_{ba}\varepsilon & H_{bb} - \varepsilon \end{vmatrix} = 0$ Secular determinant:  $\begin{vmatrix} H_{aa} - \varepsilon & H_{ab} - S_{ab} \varepsilon \\ H_{ab} - S_{ab} \varepsilon & H_{aa} - \varepsilon \end{vmatrix} = 0$ homonuclear diatomic  $\left(H_{aa}-\varepsilon\right)^2-\left(H_{ab}-S_{ab}\varepsilon\right)^2=0$ expanding  $(H_{aa} - \varepsilon) = \pm (H_{ab} - S_{ab}\varepsilon)$ Taking the root:

Yields 
$$\varepsilon_1 = \frac{H_{aa} + H_{ab}}{1 + S}$$
 and  $\varepsilon_2 = \frac{H_{aa} - H_{ab}}{1 - S}$ 



 $\begin{array}{l} \mathsf{H}_{aa} = \mathsf{H}_{bb} \ < 0 \\ \mathsf{H}_{ab} \ < 0 \end{array}$ 

Makes  $\varepsilon_2 > \varepsilon_1$ .

Substituting  $\epsilon$  in secular equations we get information about the coefficients.

and 
$$c_a = c_b$$
 for  $\varepsilon_1$   
 $c_a = -c_b$  for  $\varepsilon_2$ 



MO energy diagram |equal coefficients|.

# Substituting the energy values in;

$$c_a(H_{aa} - \varepsilon) + c_b(H_{ab} - \varepsilon S_{ab}) = 0$$
  
$$c_a(H_{ab} - \varepsilon S_{ab}) + c_b(H_{bb} - \varepsilon) = 0$$

 $c = c_1 = \frac{1}{1}$  for  $\varepsilon_1$ Gives the coefficient after normalizat

is 
$$c_a = \frac{1}{\sqrt{2-2S}}$$
 for  $\varepsilon_2$  and  $c_b = -c_a$   
tion

and the wavefunctions  $\psi_1 = c_a \phi_a + c_b \phi_b = \frac{1}{\sqrt{2+2S}} \left( \phi_a + \phi_b \right) for \quad \varepsilon_1$ and the  $\psi_2 = c_a \phi_1 - c_b \phi_2 = \frac{1}{\sqrt{2 - 2S}} (\phi_a - \phi_b) \text{ for } \varepsilon_2$ 













Symmetry classification of MOs: g symmetric (about center of molecule) u antisymmetric









#### Molecular Orbitals: Homonuclear Diatomic Molecules

Molecular orbitals are spaces in a molecule electrons could reside.

The wave function for MOs are constructed as linear combinations of atomic orbitals of the atoms involved LCAO-MO.

The extent to which each orbital from an atom contribute to the MO is related to the square of the respective coefficient. N

$$\psi_j = \sum_{i=1}^N c_{ij} \phi_i$$

The MOs have shapes and spatial extents.

The MOs of homonuclear diatomics are divided into two groups with regard to two symmetry operations. (g or u symmetry, w.r.t. center of molecule indicated)

1. Rotation about nuclear (z) axis - rotation do not change MO - σ symmetry.

2. MO has a nodal plane in the molecular axis –  $\pi$  symmetry.





Note the \* antibonding orbital; high energy counterpart of bonding orbital.



Further, the AOs involved should have <u>comparable</u> <u>energies</u> as well to form bonds. (to get a significant coefficient) in the LCAO-MO;

$$\psi_j = \sum_{i=1}^N c_{ij} \phi_i$$

# Notation: MO homonuclear diatomic molecules



TABLE 23.1	Molecular Orbitals	Used to Describe Che tomic Molecules	emical Bonding
MO Designation	Alternate	Character	Atomic Orbitals
$1\sigma_g$	$\sigma_g(1s)$	Bonding	15
$1\sigma_u^*$	$\sigma^*_u(1s)$	Antibonding	15
$2\sigma_g$	$\sigma_{g}(2s)$	Bonding	$2s(2p_{z})$
$2\sigma_{\mu}^{*}$	$\sigma_u^*(2s)$	Antibonding	$2s(2p_z)$
$3\sigma_g$	$\sigma_g(2p_z)$	Bonding	$2p_z(2s)$
$3\sigma_{\mu}^{*}$	$\sigma_u^*(2p_z)$	Antibonding	$2p_{z}(2s)$
$1\pi_{\mu}$	$\pi_u \left( 2p_x, 2p_y \right)$	Bonding	$2p_s, 2p_y$
$1\pi_g^*$	$\pi_{g}^{*}(2p_{x}, 2p_{y})$	Antibonding	$2p_x, 2p_y$

 $H_{2}$   $\left(1\sigma_{g}\right)^{2}$   $H_{2}$   $\left(1\sigma_{g}\right)^{2}$   $H_{2}$   $\left(1\sigma_{g}\right)^{2}\left(1\sigma_{u}^{*}\right)^{2}$   $H_{3}$   $\left(1\sigma_{g}\right)^{2}\left(1\sigma_{u}^{*}\right)^{2}$   $H_{3}$   $\left(1\sigma_{g}\right)^{2}\left(1\sigma_{u}^{*}\right)^{2}$   $H_{3}$   $\left(1\sigma_{g}\right)^{2}\left(1\sigma_{u}^{*}\right)^{2}$   $\left(1\sigma_{g}\right)^{2}\left(1\sigma_{u}^{*}\right)^{2}$   $\left(1\sigma_{g}\right)^{2}\left(1\sigma_{u}^{*}\right)^{2}$   $\left(1\sigma_{g}\right)^{2}\left(1\sigma_{u}^{*}\right)^{2}$   $\left(1\sigma_{g}\right)^{2}\left(1\sigma_{u}^{*}\right)^{2}$   $\left(1\sigma_{g}\right)^{2}\left(1\sigma_{u}^{*}\right)^{2}$ 



#### Minimal basis set sufficient for diatomics.









Closed-Shell Molecular Orbital Coefficients - H<sub>2</sub>

MO:		1	2	
Eigenvalues:		-0.59018	0.70055	
(ev)		-16.05961	19.06286	
1 H1 2 H2	S S	Sg+ 0.54587 0.54587	Su+ 1.24613 -1.24613	





Closed-Shell Molecular Orbital Coefficients								
мо:	1	2	3	4	5			
Energy - (eV) -4	15.50632 21.94859	-15.50495 -421.91138	-1.40847 -38.32647	-0.72753 -19.79720	-0.54859 -14.92781			
	Sg+	Su+	Sg+	Su+	Piu			
1 N1 S1 2 N1 S 3 N1 PX 4 N1 PY	0.70318 0.01286 0.00000 0.00000	-0.70282 -0.02571 0.00000 0.00000	-0.17370 0.50000 0.00000 0.00000	-0.17256 0.74662 0.00000 0.00000	0.00000 0.00000 0.62965 0.00000			
5 N1 PZ 6 N2 S1	-0.00171 0.70318	0.00924	-0.23027 -0.17370	0.25277	0.00000			
7 N2 S 8 N2 PX 9 N2 PY	0.01286 0.00000 0.00000	0.02571 0.00000 0.00000	0.50000 0.00000 0.00000	-0.74662 0.00000 0.00000	0.00000 0.62965 0.00000			
10 N2 PZ	0.00171	0.00924	0.23027	0.25277	0.0000			

MO:	6	7	8	9	10
Eigenvalues: -0.54859		-0.53025	0.26531	0.26531	1.04079
(ev) -14.92781		-14.42883	7.21946	7.21946	28.32122
	Piu	Sg+	Pig	Pig	Su+
l N1 S1	0.00000	0.06956	0.00000	0.00000	0.12483
2 N1 S	0.00000	-0.39958	0.00000	0.00000	-1.09460
3 N1 PX	0.00000	0.00000	-0.82265	0.00000	0.00000
4 N1 PY	0.62965	0.00000	0.00000	-0.82265	0.00000
5 N1 PZ	0.00000	-0.60424	0.00000	0.00000	1.16295
6 N2 S1	0.00000	0.06956	0.00000	0.00000	-0.12483
7 N2 S	0.00000	-0.39958	0.00000	0.00000	1.09460
8 N2 PX	0.00000	0.00000	0.82265	0.00000	0.00000
9 N2 PY	0.62965	0.00000	0.00000	0.82265	0.00000
10 N2 PZ	0.00000	0.60424	0.00000	0.00000	1.16295





Electronic configuration, <u>bond length</u> (pm), <u>bond order</u> and <u>bond energy</u> (kJ mol<sup>-1</sup> of Li<sub>2</sub> to  $F_2 \ [\sigma_{1s}^2 \sigma_{1s}^*^2]...$ 

	Electronic configuration	Bond length	Bond energy
Li-Li	$\sigma_{2s}^{2}$	267	110
BeBe	$\sigma_{2s}^{2} \sigma_{2s}^{*2}$	exist?	exist?
B-B	$\sigma_{2s}^{2} \sigma_{2s}^{*2} \pi_{2p}^{2}$	159	290
C=C	$\sigma_{2s}^{2} \sigma_{2s}^{*2} \pi_{2p}^{4}$	124	602
N≡N	$\sigma_{2s}^{2}\sigma_{2s}^{*}{}^{2}\pi_{2p}^{4}\sigma_{2p}^{2}$	110	942
O=0	$\sigma_{2s}^{2} \sigma_{2s}^{*2} \sigma_{2p}^{2} \pi_{2p}^{4} \pi_{2p}^{*2}$	121	494
F-F	$\sigma_{2s}^{2} \sigma_{2s}^{*2} \sigma_{2p}^{2} \pi_{2p}^{4} \pi_{2p}^{*4}$	142	155



Delta bond:



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Delta bonds ( $\delta$  bonds) are covalent chemical bonds, where four lobes of one atom is involved with four lobes of the another atom.

This overlap leads to the formation of a bonding molecular orbital with two nodal planes which contains the internuclear axis and go through both atoms.

https://en.wikipedia.org/wiki/Delta\_bond

# Molecular Orbitals: Hetero-nuclear Diatomic Molecules

e.g. HF - Energies of AOs unequal.

(mimimal) basis functions -  $\phi_{H1s}$  and  $\phi_{F2pz}$ 



Reminder: square of the coefficient is the contribution to electron probability distribution in the bond.

 $\psi_1 = c_{1H}\phi_{H1s} + c_{1F}\phi_{F2p_z}$  and  $\psi_2 = c_{2H}\phi_{H1s} + c_{2F}\phi_{F2p_z}$ 

 $\psi_{1} = c_{1H}\phi_{H1s} + c_{1F}\phi_{F2pz}$  $\int \psi_{1} * \psi_{1}d\tau = c_{1H}^{2} + c_{1F}^{2} + 2c_{1H}c_{1F}S_{HF}$ 

$c_{1H}^{2} = 0.34^{2} = 0.12$	Around H
$c_{1F}^{2} = 0.84^{2} = 0.71$	Around F
$2c_{1H}c_{1F}S_{HF} = 0.17$	Shared H and F

Clo:	sed-S	Shell Mo	lecular Orbit	al Coeffici	ents		
MO	:		1	2	3	4	5
Ei	genva	ilues:	-25.90350	-1.45986	-0.57366	-0.46312	-0.4631
		(ev)	-704.87053	-39.72485	-15,61001	-12.60214	-12.6021
			A1	A1	A1	E1x	E1y
1	F1	S1	0.99475	-0.25068	0.07827	0.00000	0.0000
2	F1	S	0.02226	0.94670	-0.41091	0.00000	0.0000
3	F1	PX	0.00000	0.00000	0.00000	1.00000	0.0000
- 4	F1	PY	0.00000	0.00000	0.00000	0.00000	1.0000
5	F1	PZ	0.00267	0.07826	0.69806	0.00000	0.0000
- 6	H1	S	-0.00534	0.15043	0.53370	0.00000	0.0000
MO	:		6				
Ei	genva	alues:	0.58984				
		(ev)	16.05025				
			A1				
1	F1	S1	0.08057				
2	F1	S	-0.51586				
3	F1	PX	0.00000				
-4	F1	PY	0.00000				
5	F1	ΡZ	-0.81644				
6	H1	S	1.05435				

Closed-Shell Molecular Orbital Coefficients MO: 1 2 3 4 5 Eigenvalues: -25.90350 -1.45986 -0.57366 -0.46312 -0.46312 (ev) -704.87053 -39.72485 -15.61000 -12.60214 -12.60214

			Al	A1	Al	Elx	Ely
1	F1	S1	0.99475	-0.25068	0.07827	0.00000	0.00000
2	F1	s	0.02226	0.94670	-0.41091	0.00000	0.00000
3	F1	РX	0.00000	0.00000	0.00000	1.00000	0.00000
4	F1	ΡY	0.00000	0.00000	0.00000	0.00000	1.00000
5	F1	ΡZ	0.00267	0.07826	0.69806	0.00000	0.00000
6	H1	s	-0.00534	0.15043	0.53370	0.00000	0.00000
					in phas	е	







## Molecular Electrostatic Potential (MEP)

The potential felt by a test charge (unit positive) at various points in the molecule.

MEP is calculated by considering the positively charged nuclear framework and the negatively charged electron cloud of the molecule.

A negative MEP signifies attraction of a unit positive charge, therefore a negatively charged point in the molecule.

A positive MEP signifies repulsion of a unit positive charge, therefore a positively charged point in the molecule.

$$\phi(r) = \frac{q}{4\pi\varepsilon_0 r}$$
  
$$\phi_{nuclei}(x_1, y_1, z_1) = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i}$$

 $\rho(x, y, z) = -e \int \dots \int (\psi(x, y, z; x_1, y_1, z_1; \dots; x_n, y_n, z_n))^2$ 

 $\times dx_1 dy_1 dz_1 \dots dx_n dy_n dz_n$ 

$$\phi(x_1, y_1, z_1) = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i} - e \iiint \frac{\rho(x, y, z)}{4\pi\varepsilon_0 r_e} \, dx \, dy \, dz$$

#### **Electrostatic Potential Surfaces**

Molecular electrical potential surfaces, illustrate the charge distributions of molecules three dimensionally.

The surface is a constant electron density surface.

The potential at the surface is mapped on to the surface. The surface (solid/transparent or mesh) is color coded according to the electrical potential.

Knowledge of the charge distributions can be used to determine how molecules interact with one another.





4/28/2017







