Exam 3 – Counting and Probability

Math 102–Fall 2006 Name Key

(12) 1. Complete each of the following with the appropriate word, phrase, or symbols.

(a) The set of all possible outcomes of an experiment is called a Sample space.

(b) The probability of an impossible event is 0.

(c) A subset of the sample space is called an event.

(d) $\dfrac{\text{probability of } A \text{ will occur}}{\text{probability that } A \text{ will not occur}} =$ odds in favor.

(e) $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

(10) 2. Find the value of each.

(a) $0! = 1$

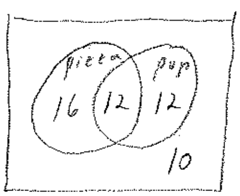
(b) $\dfrac{7!}{4!} = 7 \cdot 6 \cdot 5 = 210$

(c) $C(6,4) = \dfrac{6!}{4!2!} = \dfrac{\overset{3}{\cancel{6}} \cdot 5}{\cancel{2} \cdot 1} = 15$

(d) $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

(e) $P(6,4) = \dfrac{6!}{(6-4)!} = \dfrac{6!}{2!} = 6 \cdot 5 \cdot 4 \cdot 3 = 360$

(12) 3. In a survey of fifty people, the following information was obtained:
twelve eat pizza and drink pop, twenty-eight eat pizza, and twenty-four drink pop.



(a) Find the probability that a person randomly chosen only eats pizza. $\dfrac{16}{50} = \dfrac{8}{25}$

(b) Find the probability that a person randomly chosen eats pizza or drinks pop. $\dfrac{40}{50} = \dfrac{4}{5}$

(c) Find the probability that a person randomly chosen neither eats pizza nor drinks pop.

$\dfrac{10}{50} = \dfrac{1}{5}$

(18) 4. On a single random draw from a shuffled standard deck of 52 cards, find

(a) the odds in favor of drawing a jack. $\dfrac{1}{13} \div \dfrac{12}{13} = \dfrac{1}{13} \cdot \dfrac{13}{12} = \dfrac{1}{12}$ $1:12$

(b) the odds against drawing a heart. $\dfrac{3}{4} \div \dfrac{1}{4} = \dfrac{3}{4} \cdot \dfrac{4}{1} = \dfrac{3}{1}$ $3:1$

(c) the probability of drawing a ten and a diamond. $\dfrac{1}{13} \cdot \dfrac{1}{4} = \dfrac{1}{52}$

(d) the probability of drawing a ten or a diamond. $\dfrac{1}{13} + \dfrac{1}{4} - \dfrac{1}{52} = \dfrac{4}{52} + \dfrac{13}{52} - \dfrac{1}{52} = \dfrac{16}{52} = \dfrac{4}{13}$

(e) the probability of drawing a ten and an ace. 0

(f) the probability of drawing a ten or an ace. $\dfrac{1}{13} + \dfrac{1}{13} - 0 = \dfrac{2}{13}$

(5) 5. In a game where you flip two coins, you win one dollar when both tails show and two dollars when both heads show; otherwise, you lose. What is a fair price to pay to play?

$EV = \frac{1}{4}(1) + \frac{1}{4}(2) + \frac{1}{2}(0) = \frac{3}{4} = 0.75$

A fair price to pay is 75¢.

(5) 6. Assume a game where two fair dice are rolled. If a total of six, seven or eight comes up, a person wins \$5; if a two or twelve comes up, a person wins \$3; otherwise, the person loses. What is the expected value of the game?

$EV = \frac{16}{36}(5) + \frac{2}{36}(3) + \frac{18}{36}(0) = \frac{40}{18} + \frac{3}{18} = \frac{43}{18} =$

The expected value is $\$2.38\frac{8}{9}$.

$$\begin{array}{r} 2.38 \\ 18\overline{)43.00} \\ \underline{36} \\ 70 \\ \underline{54} \\ 160 \\ \underline{144} \\ 16 \end{array}$$

(12) 7. A bag contains five red marbles, three green marbles, and two blue marbles. When three marbles are randomly drawn (1) with replacement, and (2) without replacement, find the probability that

(a) all three are blue

(1) $\frac{2}{10} \cdot \frac{2}{10} \cdot \frac{2}{10} = \frac{1}{125}$ (2) $\frac{2}{10} \cdot \frac{1}{9} \cdot \frac{0}{8} = 0$

(b) first blue, second red, and third green

(1) $\frac{2}{10} \cdot \frac{5}{10} \cdot \frac{3}{10} = \frac{3}{100}$ (2) $\frac{2}{10} \cdot \frac{5}{9} \cdot \frac{3}{8} = \frac{1}{24}$

(6) 8. Given two events A and B where $P(A) = \frac{1}{5}$, $P(B) = \frac{7}{10}$, and $P(A \cap B) = \frac{1}{10}$, find $P(A \cup B)$.

$P(A \cup B) = \frac{1}{5} + \frac{7}{10} - \frac{1}{10} = \frac{2}{10} + \frac{7}{10} - \frac{1}{10} = \frac{8}{10} = \frac{4}{5}$

(5) 9. A club of ten people elects a president, vice-president, treasurer, and secretary. If a person can hold only one office, in how many ways can a set of officers be chosen?

${}_{10}P_4 = \frac{10!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$

(5) 10. How many distinct "words" can be formed using all the letters in *DEFENSE*?

$\frac{7!}{\underset{E}{3!}\,1!\,1!\,1!\,1!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840$

(5) 11. How many ways can a committee of three be chosen in a club of ten people?

${}_{10}C_3 = \frac{10!}{3!\,7!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$

(5) 12. How many ways can a committee of four men and three women be chosen from a club of twelve men and eight women?

${}_{12}C_4 \cdot {}_8C_3 = \frac{12!}{4!\,8!} \cdot \frac{8!}{3!\,5!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$

$= 11 \cdot 5 \cdot 9 \cdot 8 \cdot 7 = 27{,}720$

(5) E.C. Find the probability of being dealt a full house from a standard deck.

$$\frac{{}_{13}C_2 \cdot {}_2C_1 \cdot {}_4C_3 \cdot {}_4C_2}{{}_{52}C_5}$$