Exam 3 — Counting and Probability

Use proper notation and show your work. Name Key

Counting

1. Calculate each value.

(a) $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

(b) $\frac{10!}{7!} = 10 \cdot 9 \cdot 8 = 720$

(c) ${}_6P_2 = 6 \cdot 5 = 30$

(d) ${}_{10}C_3 = \frac{10!}{3!7!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 120$

2. The Equestrian Club has eight members. If the club wants to select a president, vice-president, and treasurer (all of whom must be different), in how many ways can this be done?

${}_8P_3 = 8 \cdot 7 \cdot 6 = 336$

There are 336 different possible arrangements for the election of officers.

3. In how many ways can Elaine build a music system consisting of amplifier, speakers, tape deck, and CD player, if she can select from four amplifiers, eight types of speakers, three tape decks, and five CD players?

$4 \cdot 8 \cdot 3 \cdot 5 = 480$

There are 480 different possible music systems.

4. Find the number of possible distinct passwords that can be formed by rearranging all the letters in the word *sassafras*?

$\frac{9!}{4!3!1!1!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 9 \cdot 8 \cdot 7 \cdot 5 = 2520$

There are 2520 distinct passwords possible.

5. A class of 12 students has its pictures taken in groups of five. How many different group pictures are possible?

${}_{12}C_5 = \frac{12!}{5!7!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 11 \cdot 9 \cdot 8 = 792$

There are 792 different group pictures that can be taken.

Probability

6. Select a card randomly from a standard 52-card deck.
   (a) What is the probability that a heart is drawn?

$\frac{13}{52} = \frac{1}{4}$

The probability of drawing a heart is $\frac{1}{4}$.

   (b) What are the odds against drawing a face card?

$\frac{10}{13} \div \frac{3}{13} = \frac{10}{13} \cdot \frac{13}{3} = \frac{10}{3}$ 10 : 3

The odds against drawing a face card is 10 to 3.

   (c) What is the probability that it is a five or a red card?

$\frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$

The probability of drawing a five or a red card is $\frac{7}{13}$.

7. A ball is randomly selected from a jar containing 70 red balls, 20 green balls, and 10 blue balls. What is the probability that a non-red ball is chosen?

$\frac{30}{100} = \frac{3}{10}$

The probability a non-red ball is chosen is $\frac{3}{10}$.

8. If $P(A \cup B) = 0.85$, $P(B) = 0.40$, and $P(A) = 0.55$, find $P(A \cap B)$.

$0.85 = 0.4 + 0.55 - P(A \cap B)$

$0.85 = 0.95 - P(A \cap B)$

$-0.1 = -P(A \cap B)$

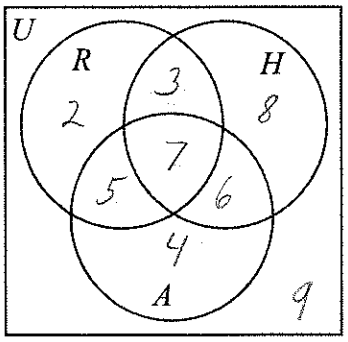
$0.1 = P(A \cap B)$

9. If four coins are flipped, what is the probability that all the coins are heads?

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

The probability all the coins are heads is $\frac{1}{16}$.

10. MusiChan.com surveyed a group of subscribers regarding which online music channels they use on a regular basis. The following information summarizes their answers:
7 listened to rap, heavy metal, and alternative rock; 10 listened to rap and heavy metal;
13 listened to heavy metal and alternative rock; 12 listened to rap and alternative rock;
17 listened to rap; 24 listened to heavy metal; 22 listened to alternative rock; and
9 listened to none of these three channels.
Find the probability of randomly selecting a person for each event.



(a) Probability the person listens to rap or alternative rock.

$\frac{27}{44}$

(b) Probability the person listens to heavy metal only.

$\frac{8}{44} = \frac{2}{11}$

(c) Probability the person listens to neither rap nor heavy metal.

$\frac{13}{44}$

(d) Probability the person listens to rap and heavy metal, but not to alternate rock.

$\frac{3}{44}$

11. We draw three cards from a standard 52-card deck. Find the probability of drawing three spades.

(a) with replacement

$\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{64}$

(b) without replacement

$\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \frac{1}{4} \cdot \frac{4}{17} \cdot \frac{11}{50}$

$= \frac{11}{850}$

12. Two balls are drawn from an urn containing five red balls, three green balls, and nine blue balls. Find the probability the first ball is blue and the second ball is red.

(a) with replacement

$\frac{9}{17} \cdot \frac{5}{17} = \frac{45}{289}$

(b) without replacement

$\frac{9}{17} \cdot \frac{5}{16} = \frac{45}{272}$

13. A game is played where a single die is rolled. If an odd number comes up, you win the number of dollars showing on the die. If an even number comes up, you lose the number of dollars showing on the die. Determine a "fair" price to pay to play.

$\frac{1}{6}(1) + \frac{1}{6}(3) + \frac{1}{6}(5) + \frac{1}{6}(-2) + \frac{1}{6}(-4) + \frac{1}{6}(-6)$

$= \frac{1}{6} + \frac{3}{6} + \frac{5}{6} - \frac{2}{6} - \frac{4}{6} - \frac{6}{6} = \frac{-3}{6} = -\frac{1}{2}$

The expected value is a loss of 50¢.

14. Assume that the probability of a 25-year-old male living to age 26, based on mortality tables, is 0.98. If a $1000 one-year term life insurance policy on a 25-year-old male costs $27.50, what is the expected value?

$0.98(-27.50) + 0.02(972.5)$

15. Assume that we draw a five-card hand from a standard 52-card deck. What is the probability that all cards are diamonds?

$$\frac{{}_{13}C_5}{{}_{52}C_5} = \frac{\frac{13!}{8!5!}}{\frac{52!}{5!47!}} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{5 \cdot 4 \cdot 3 \cdot 2} \cdot \frac{5 \cdot 4 \cdot 3 \cdot 2}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}$$

$$= \frac{33}{4 \cdot 17 \cdot 5 \cdot 49 \cdot 4} = \frac{33}{66,640}$$