

Exam 1 — Sets

Use proper notation and show your work.

Name Key

1. Use the list/roster method to express the set of the natural numbers from 10 to 15, inclusive.

$$\{10, 11, 12, 13, 14, 15\}$$

2. Use set-builder notation to express the set  $\{m, u, s, i, c\}$ .

$$\{x \mid x \text{ is a letter in music.}\}$$

3. Determine whether each set is well-defined. If it is not well-defined, explain why.

- (a) The set of daily high temperatures in Fahrenheit measured at Hector International Airport in Fargo, North Dakota during the year 2010

Well defined

- (b)  $\{x : x \text{ is a ferocious animal.}\}$

Not well-defined, the word ferocious is a relative term.

4. Determine whether each statement is true or false.

false (a)  $3 \in \{2, 4, 6, 8\}$

false (b)  $0 \in \emptyset$

false (c)  $\{1\} \in \{1, 2, 3\}$

true (d)  $\{2\} \subseteq \{1, 2, 3\}$

true (e)  $\{1, 3, 5, 7, 9\} = \{1, 5, 9, 3, 7\}$

false (f)  $\{1, 3, 5, 9\} \subset \{1, 3, 5, 9\}$

true (g)  $\emptyset \subseteq \{1, 3, 5\}$

true (h)  $\{1, 2, 3, 4, 5\}$  and  $\{a, e, i, o, u\}$  are equivalent sets.

true (i) If  $A \subseteq B$  and  $B \subseteq A$ , then  $A = B$ . true (j)  $X \subseteq (X \cup Y)$

true (k)  $\{x : x \text{ is a letter in song.}\}$  and  $\{x : x \text{ is a letter in songs.}\}$  are equivalent.

5. Determine the cardinality,  $n(A)$ , for each set.

(a)  $A = \{1, 3, 5, 7, \dots, 11\}$

$$n(A) = 6$$

(b)  $A = \{x : x \text{ is a letter in the word Mississippi.}\}$

$$= \{M, i, s, p\}$$

$$n(A) = 4$$

6. Describe each of the following sets as either finite or infinite.

(a)  $\{x : x \text{ is a natural number greater than } 58.\}$

*infinite*

(b)  $\{x : x \text{ is a person who has walked on the moon.}\}$

*finite*

7. List all the two element subsets of  $\{1, 2, 3\}$ .

$\{1, 2\}, \{1, 3\}, \{2, 3\}$

8. If a set  $A$  has 5 elements, how many subsets does  $A$  have?

$$2^5 = 32$$

9. Show the sets  $\{1, 2, 3, 4\}$  and  $\{\text{left, right, up, down}\}$  are equivalent.

1	2	3	4
1	1	1	1
left	right	up	down

10. Show  $\{7, 14, 21, 28, 35, \dots\}$  is infinite by placing it in a one-to-one correspondence with a proper subset of itself. Be sure to show the pairing of the general terms in the sets.

7	14	21	28	35	...	$7n$
1	1	1	1	1		1
14	21	28	35	42	...	$7n+7$

11. Show  $\left\{\frac{1}{6}, \frac{2}{7}, \frac{3}{8}, \frac{4}{9}, \dots\right\}$  has cardinal number  $\aleph_0$  by establishing a one-to-one correspondence between it and the set of natural numbers.

$\frac{1}{6}$	$\frac{2}{7}$	$\frac{3}{8}$	$\frac{4}{9}$	...	$\frac{n}{n+5}$
1	1	1	1		1
1	2	3	4	...	$n$

12. Let  $U = \{1, 2, 3, \dots, 10\}$ ,  $A = \{1, 3, 5, 7, 9\}$ ,  $B = \{1, 2, 3, 4, 5, 6\}$ , and  $C = \{2, 4, 6, 7, 8\}$ . Perform the indicated operations.

(a)  $A \cap B$

$= \{3, 5\}$

(b)  $B \cup C$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(c)  $A \cap (B \cup C)$

$= \{1, 3, 5, 7\}$

(d)  $(A - B) \cap (A - C)$

$= \{7, 9\} \cap \{1, 3, 5, 9\}$   
 $= \{9\}$

(e)  $A' \cap (B \cup C')$

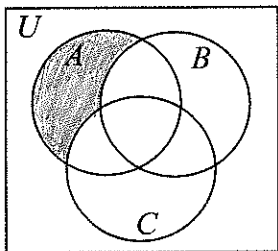
$= \{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5, 6, 9, 10\}$

$= \{2, 4, 6, 10\}$

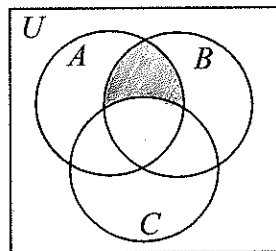
(f)  $A \cup \emptyset = A$

13. Represent each set on the Venn diagram.

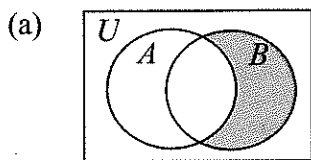
(a)  $A - (B \cup C)$



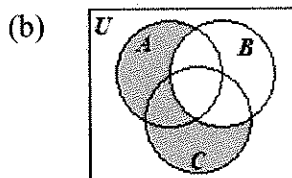
(b)  $(A \cap B) - C$



14. Describe the shaded region using set theory notation.



$B - A$   
 or  $A' \cap B$



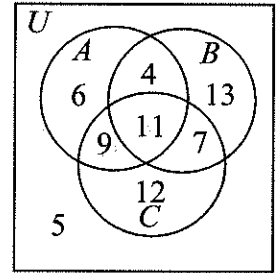
$(A \cup C) - B$   
 or  $(A \cup C) \cap B'$

15. Find the following using the given Venn diagram.

(a)  $n(C') = 6 + 4 + 13 + 5 = 28$

(b)  $n((A \cup B) \cap C) = 9 + 11 + 7 = 27$

(c)  $n(A \cap C) = 9 + 11 = 20$



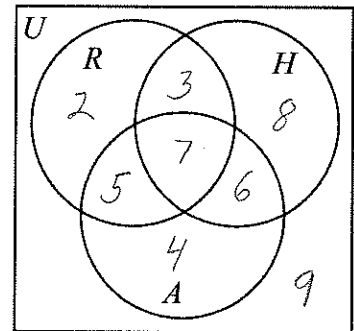
16. MusiChan.com surveyed a group of subscribers regarding which online music channels they use on a regular basis. The following information summarizes their answers:

- 7 listened to rap, heavy metal, and alternative rock; 10 listened to rap and heavy metal;
- 13 listened to heavy metal and alternative rock; 12 listened to rap and alternative rock;
- 17 listened to rap; 24 listened to heavy metal; 22 listened to alternative rock; and
- 9 listened to none of these three channels.

(a) How many people were surveyed?

$$n(U) = 2 + 3 + 8 + 5 + 7 + 6 + 4 + 9 = 44$$

Forty-four people were surveyed.



(b) How many people listened to rap or alternative rock?

$$n(R \cup A) = 2 + 3 + 5 + 7 + 4 + 6 = 27$$

Twenty-seven people listened to rap or alternative rock.

(c) How many listened to heavy metal only?

$$n(H - (R \cup A)) = 8$$

Eight listened to heavy metal only.