

Percent Problems

Interpret each of the following statements.

A mortgage interest rate is 6.3%.

A CD earns 2.85%.

Many people tip 15%.

Each of the problems involves percentages, which are a very common method for stating ratios involving monetary values. The mortgage interest rate means that a person must pay \$6.30 for every \$100 borrowed, which is a ratio of 6.3 to 100. The CD earns \$2.85 for every \$100 put into the CD, which is a ratio of 2.85 to 100. A tip of 15% means that the people tip \$15 for every \$100 paid for a meal, which is a ratio of 15 to 100 or 3 to 20.

Remember, in our last lesson, we stated that a *ratio* is a comparison of the relative size of two numerical quantities. A percent is a special type of ratio where the second value is always 100.

Definition: A *percent* is ratio where n percent, denoted as $n\%$, is the ratio $n : 100$. That is, a ratio compares numerical quantities in terms of the number of parts out of 100.

Examples: Rewrite $\frac{1}{3}\%$ and $46\frac{2}{7}\%$ as common fractions.

Again we use the definition of percent and write the ratio in fraction form, but note that the numerator contains common fractions or mixed numbers. We use an appropriate multiplier and then simplify.

$$\frac{1}{3}\% = \frac{\frac{1}{3}}{100} = \frac{\frac{1}{3} \cdot 3}{100 \cdot 3} = \frac{1}{300}$$

$$46\frac{2}{7}\% = \frac{46\frac{2}{7}}{100} = \frac{\frac{324}{7} \cdot 7}{100 \cdot 7} = \frac{324}{700} = \frac{81(4)}{175(4)} = \frac{81}{175}$$

Note. Here are some basic percent-fraction equivalences that should be memorized for use with mental estimation.

$$10\% = \frac{1}{10}$$

$$20\% = \frac{1}{5}$$

$$25\% = \frac{1}{4}$$

$$12\frac{1}{2}\% = \frac{1}{8}$$

$$33\frac{1}{3}\% = \frac{1}{3}$$

$$16\frac{2}{3}\% = \frac{1}{6}$$

$$30\% = \frac{3}{10}$$

$$40\% = \frac{2}{5}$$

$$50\% = \frac{1}{2}$$

$$37\frac{1}{2}\% = \frac{3}{8}$$

$$66\frac{2}{3}\% = \frac{2}{3}$$

$$83\frac{1}{3}\% = \frac{5}{6}$$

$$70\% = \frac{7}{10}$$

$$60\% = \frac{3}{5}$$

$$75\% = \frac{3}{4}$$

$$62\frac{1}{2}\% = \frac{5}{8}$$

$$90\% = \frac{9}{10}$$

$$80\% = \frac{4}{5}$$

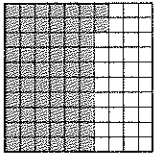
$$100\% = 1$$

$$87\frac{1}{2}\% = \frac{7}{8}$$

$$\text{Especially } 1\% = \frac{1}{100}.$$

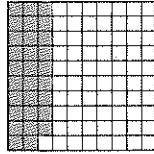
1. Write the percentage represented by each of the following diagrams:

(a)



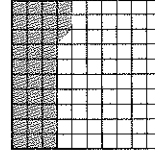
62%

(b)



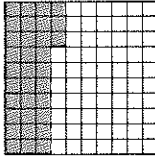
29%

(c)



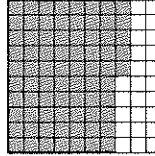
$32\frac{1}{2}\%$

(d)



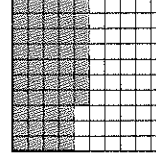
33%

(e)



75%

(f)



47%

2. Solve each percent problem

(a) What is 10% of 250? ²⁵

$$n = \frac{1}{10} \cdot \frac{250}{1}$$

$$= 25$$

(b) 15 is what percent of 90?

$$15 = n(90)$$

$$\frac{15}{90} = n$$

$$16\frac{2}{3}\%$$

$$\begin{array}{r} 16\frac{2}{3} \\ 90 \overline{) 15.00} \\ \underline{-90} \\ 600 \\ \underline{-540} \\ 60 \end{array}$$

(c) 18% of what is 900?

$$0.18n = 900$$

$$n = \frac{900}{0.18}$$

$$n = 5000$$

$$\begin{array}{r} 5000 \\ 0.18 \overline{) 90000} \\ \underline{-90} \\ 0 \end{array}$$

(d) How much is 25% of 120?

$$n = \frac{1}{4} \cdot \frac{120}{1}$$

$$= 30$$

(e) 86% of what is 215?

$$0.86n = 215$$

$$n = \frac{215}{0.86}$$

$$= 250$$

$$\begin{array}{r} 250 \\ 0.86 \overline{) 215.00} \\ \underline{-172} \\ 430 \\ \underline{-430} \\ 0 \end{array}$$

(e) What percent of 70 is 5?

$$n(70) = 5$$

$$n = \frac{5}{70}$$

$$7\frac{1}{7}\%$$

$$\begin{array}{r} 0.07\frac{1}{7} \\ 70 \overline{) 5.00} \\ \underline{490} \\ 10 \end{array}$$

(g) 8% of 30 is how much?

$$0.08(30) = n$$

$$n = 2.4$$

(h) 54 is 90% of what?

$$54 = 0.9n$$

$$n = 60$$

$$\begin{array}{r} 60 \\ 0.9 \overline{) 54.0} \\ \underline{-54} \\ 0 \end{array}$$

(i) What is 110% of 50?

$$n = 1.1(50) \\ = 55$$

(j) 10 is 10% of what?

$$10 = \frac{1}{10} n \\ n = 100$$

(k) 30% of 93 is how much?

$$0.3(93) = n \\ \begin{array}{r} 93 \\ 0.3 \\ \hline 27.9 \end{array}$$

(l) 85% of what is 340?

$$0.85n = 340 \\ n = \frac{340}{0.85} \\ = 400$$

$$\begin{array}{r} 400. \\ 0.85 \overline{) 340.00} \\ \underline{-340} \end{array}$$

(m) 30 out of 40 is what percent?

$$\frac{30}{40} = n \\ n = 75\%$$

(n) What percent of 50 is 23?

$$n(50) = 23 \\ n = \frac{23}{50} = \frac{46}{100} \\ 46\%$$

(o) How much is 28% of 50?

$$n = 0.28(50) \\ \begin{array}{r} 0.28 \\ 50 \\ \hline 14.00 \end{array} \quad n = 14$$

(p) 25 is what percent of 80?

$$25 = n(80) \\ \frac{25}{80} = n$$

$$31\frac{1}{4}\%$$

$$\begin{array}{r} 0.31\frac{25}{80} \\ 80 \overline{) 25.00} \\ \underline{-240} \\ 100 \\ \underline{80} \\ 20 \end{array}$$

3. 40% of all television programs include scenes of excessive violence in at least one episode per season. If 500 television shows were examined in researching this statistic, how many of them were found to have at least one episode of excessive violence in the season?

$$0.40(500) = 40(5) = 200$$

Two hundred of the episodes have at least one episode of excessive violence.

4. Of the 500 people on the cruise 429 said that if they were to book another cruise, they would book with the same cruise company. What percent is this?

$$n(500) = 429$$

$$n = \frac{429}{500} = \frac{858}{1000} = \frac{85.8}{100} = 85.8\%$$

This is 85.8% of the people on the cruise.

5. For a child, the proper dosage of a medicine is 25% of the adult-strength dosage. This means that a child gets 5 mg of the active ingredient. How many mg of the active ingredient are in an adult-strength dosage?

$$\frac{1}{4}n = 5$$

$$n = 5(4) = 20$$

There are 20 mg of the active ingredient in the adult-strength dosage.

6. A quality control study found that 0.05% of the gizmos being manufactured were defective. That means that out of 10,000 gizmos, how many are expected to be defective?

$$0.0005(10000) = 5$$

Five of the gizmos were defective.

7. Forty-five of the cats in the animal shelter this week were adopted. This represented 75% of all the cats in the shelter. How many cats were in the shelter this week?

$$\frac{3}{4}c = 45$$

$$c = \frac{45}{\frac{3}{4}} = \frac{45 \cdot 4}{3} = 60$$

There were 60 cats in the shelter this week.

8. Crystal missed 12 out of 88 problems on her final exam. What was her percent score?

$$\frac{88-12}{88} = \frac{76}{88} = \frac{19}{22}$$

$$\begin{array}{r} 0.86 \\ 22 \overline{) 19.00} \\ \underline{-176} \\ 140 \\ \underline{-132} \\ 8 \end{array}$$

Crystal's score was $86\frac{4}{11}\%$.

9. An orthopedic surgeon estimates that 70% of all of the patients referred to her actually need back surgery. If she saw 300 cases this month, how many would she expect to actually need surgery?

$$70\% \text{ of } 300$$

$$70(3) = 210$$

The orthopedic surgeon would expect 210 to need surgery.

10. A charitable organization spent \$2940 on administrative expenses last year. This amount is 12% of the total funds it collected. Find the total amount of money the organization collected last year.

$$0.12n = 2940$$

$$n = \frac{2940}{0.12}$$

$$0.12 \overline{) 294000} \\ \underline{24} \\ 54 \\ \underline{48} \\ 60 \\ \underline{60} \\ 0$$

The organization collected \$24,500.

11. Suppose that you used to make \$8 an hour at your part time job. Find your new hourly wage after getting a 7% pay raise.

$$8 + 0.07(8) \\ = 8 + 0.56 \\ = 8.56$$

Your new hourly wage is \$8.56.

12. Twelve marbles are placed in a paper bag: 4 are red, 6 are blue, and 2 are yellow. One marble is drawn from the bag at random. What is the probability that the marble is yellow? Give your final answer as a percent.

$$\frac{2}{12} = \frac{1}{6} = 16\frac{2}{3}\%$$

The probability the marble is yellow is $16\frac{2}{3}\%$.

13. Mary went panning for gold during a visit to Colorado last summer. It is estimated that about 0.5% of all the silt in the stream she panned in is gold. How many grams of silt would she need to pan in order to obtain one gram of gold?

$$0.005n = 1$$

$$n = \frac{1}{0.005} = \frac{1000}{5} = 200$$

Mary would need to pan 200 grams of silt.

14. In the 2009 U.S. Senate, there were 58 Democrats, 40 Republican, and 2 independent senators. Consider the 2 independent senators as Democrats for this problem, since they caucus and usually vote with the Democrats. Suppose that a bill needs a simple majority (51 votes) to pass. Thirty percent of all Democrat senators and 85% of all Republican senators vote for the bill. (a) How many voted for the bill? (b) Did the bill pass?

$$0.3(60) + 0.85(40)$$

$$= 18 + 34$$

$$= 52$$

Fifty-two voted for the bill.

The bill passed.

15. In a time management project, Hector discovers that he spends 9 hours each day playing computer games. What percent of each day is spent playing computer games?

$$\frac{9}{24} = \frac{3}{8} = 37\frac{1}{2}\%$$

Hector spent $37\frac{1}{2}\%$ of each day playing computer games.

16. After a severe car wreck, the disability insurance company concludes that Avery is 30% disabled and will pay 30% of his regular monthly salary to him as his monthly benefit. The company pays him \$450 each month as his disability benefit. What was his monthly income before the accident?

$$0.3n = 450$$

$$n = \frac{450}{0.3} = \frac{4500}{3} = 1500$$

Avery's monthly income was \$1,500 before the accident.

17. (a) Suppose you and four of your friends decide to split an extra large pizza. What percentage of the pizza do you each get if you divide it evenly?

$$\frac{100}{5} = \frac{100}{5} = 20$$

You each get 20% of the pizza.

- (b) You discover a pizza parlor that has a manager's special that gives a free order of breadsticks with the purchase of an extra large pizza. One order consists of twelve breadsticks. If one of your friends only wants two breadsticks and the rest are divided evenly, what percentage of the breadsticks do you get?

$$\frac{12-2}{4} = \frac{10}{4} = 2.5$$

$$12 \overline{) 2.50} \\ \underline{24} \\ 10$$

$$\frac{2.5}{12} = 20\frac{5}{6}\%$$

You get $20\frac{5}{6}\%$ of the breadsticks.

18. Last week, 137 people took their driver's test for the first time. Of these, 110 passed the test and received their license. What percentage of the people who took the test received their license? What percentage did not receive their license?

$$\frac{110}{137}$$

$$137 \overline{) 110.00} \\ \underline{1096} \\ 40$$

There were $80\frac{40}{137}\%$ of the people received their license and $19\frac{57}{137}\%$ who did not.

Monthly Budgeted Expenses						
Housing	Food	Transportation	Utilities	Taxes	Entertainment	Total
\$500	\$300	\$600	\$150	\$200	\$250	\$2000

19. Use the table above to find the following percentages:

(a) The percentage of your monthly budget that is spent on housing.

$$\frac{500}{2000} = \frac{1}{4} = 25\% \quad \text{You spent } 25\% \text{ on housing.}$$

(b) The percentage of your monthly budget that is spent on Entertainment.

$$\frac{250}{2000} = \frac{1}{8} = 37\frac{1}{2}\% \quad \text{You spent } 37\frac{1}{2}\% \text{ on entertainment.}$$

(c) The percentage of your monthly budget that is spent on taxes and utilities.

$$\frac{350}{2000} = \frac{7}{40} \quad \begin{array}{r} 40 \overline{) 7.00} \\ \underline{40} \\ 300 \\ \underline{280} \\ 20 \end{array} \quad \text{You spent } 17\frac{1}{2}\% \text{ on taxes and utilities.}$$

20. Use the "percent of" short cut to find the following.

(a) 10% of 250

$$\frac{1}{10} \cdot \frac{250}{1} = 25$$

(b) 20% of 470

$$\begin{aligned} 20(47) \\ = 2(47) \\ = 94 \end{aligned}$$

(c) 15% of 80

$$\begin{aligned} 10\% \text{ of } 80 = 8 \quad 5\% \text{ of } 80 = 4 \\ 12 \end{aligned}$$

(d) 18% of 60

$$\begin{array}{r} 0.18 \\ 60 \\ \hline 10.80 \end{array} \quad 10.8$$

(e) 28% of 50

$$\begin{aligned} 28\% \text{ of } 100 = 28 \\ 14 \end{aligned}$$

(f) 16% of 25

$$\begin{aligned} 16\% \text{ of } 100 = 16 \\ 16\% \text{ of } 25 = \frac{1}{4}(16) = 4 \end{aligned}$$

(g) 67% of 200

$$\begin{aligned} 67(2) \\ = 134 \end{aligned}$$

(h) 6% of 50

$$\begin{aligned} 6\% \text{ of } 100 = 6 \\ 3 \end{aligned}$$

(j) 59% of 100

$$59$$

(k) 16% of 25

$$\text{see (f)}$$

(l) 67% of 200

$$\text{see (g)}$$

(m) 6% of 50

$$\text{see (h)}$$

21. Solve each problem.

(a) 12% of 50

$$12\% \text{ of } 100 = 12$$

6

(b) 20 is what percent of 400?

$$20 = n(400)$$

$$n = \frac{20}{400} = \frac{5}{100} = 5\%$$

(c) 18% of what is 900?

$$0.18n = 900$$

$$n = \frac{900}{0.18} = \frac{90000}{18}$$

$$= \frac{10000}{2} = 5000$$

(d) What percent of 400 is 12?

$$n(400) = 12$$

$$n = \frac{12}{400} = \frac{3}{100}$$

3%

(e) 15% of 80 is how much?

$$0.15(80) = n$$

$$12 = n$$

(f) 400 is 25% of how much?

$$400 = \frac{1}{4}n$$

$$n = 4(400) = 1600$$

22. A recent survey found that 44 million people in the U.S. do not have health insurance. Of those, 30% are between the ages of 18 and 24. Find the number of people in the U.S. between 18 and 24 who do not have health insurance.

$$\begin{array}{r} 44 \\ 0.3 \\ \hline 13.2 \end{array}$$

There are 13.2 million people between 18 and 24 who do not have health insurance.

23. Recently SCE laid off 11% of its workers. What percent of the workers were not laid off?

$$100\% - 11\% = 89\%$$

SCE did not lay off 89% of its workers.

24. Raina received a 7% raise. Her new salary is what percent of her previous salary?

$$100\% + 7\% = 107\%$$

Raina's new salary is 107% of her previous salary.

25. Fergie discovers that she is spending a total of \$760 per month on room and board, and this is 95% of her monthly income.

(a) What is her monthly income?

$$0.95n = 760$$

$$n = \frac{760}{0.95}$$

$$\begin{array}{r} 800.00 \\ 0.95 \overline{) 760.00} \\ \underline{-760} \\ 0000 \end{array}$$

Fergie's monthly income is \$800.

(b) How much does she have left to spend on other things each month?

$$800 - 760 = 40$$

Fergie has \$40 to spend on other things.

26. The month of June has 30 days. It rained 40% of the days in the month of June. It rained 12 days in June.

$$0.4(30) = 12.$$

The paragraph is correct.

27. Mountain Gorillas in Africa are an endangered species. According to the Census Report published by the Bwindi Impenetrable National Park in Uganda (one of only two places mountain gorillas are still found living in the wild), the population of mountain gorillas in the park at the 2002 census was 320. In the 2006 census the population was 380. By what percent has the population increased between 2002 and 2006?

$$320n = 380 - 320$$

$$n = \frac{60}{320} = \frac{3}{16}$$

$$\begin{array}{r} .1875 \\ 16 \overline{) 3.00} \\ \underline{16} \\ 140 \\ \underline{128} \\ 12 \end{array}$$

The population increased by $18\frac{3}{4}\%$.

28. The price on a brand new GoldWing Motorcycle is reduced 18% because it is a 2008 model and still has not sold. What percent of the original price will Peter pay if he buys this motorcycle on sale?

$$100\% - 18\% = 82\%$$

Peter would pay 82% of the original price.

29. PeeWee is earning \$5.60 per hour stocking shelves at his father's store. He has done such a good job that his father gives him a 5% raise. Now how much is PeeWee earning per hour?

$$5.60 + 0.05(5.60) = 5.88$$

$$\begin{array}{r} 5.60 \\ 0.05 \\ \hline 0.2800 \end{array}$$

PeeWee's would earn \$5.88 per hour.

30. Research shows that a certain migraine medicine has a rare but serious (often fatal) side effect in 0.0035% of those who take the medication. If 200,000 people are currently taking this medicine, how many would we expect to suffer from this complication?

$$0.000035(200000) = 3.5(2) = 7$$

We would expect 7 people to suffer this complication.

31. I have a coupon for one "Magnificent Seven" breakfast at Perkins. With the coupon the breakfast costs \$2.00 less than the regular price of the breakfast, which is \$4.99. I also have a 20% off entire purchase coupon.

- (a) If I am only purchasing one "Magnificent Seven" breakfast, which coupon should I use to get the better price? (Only one coupon can be used per visit). Show any computation and explain your reasoning.

$$\begin{array}{r} 4.99 \\ 0.2 \\ \hline 0.998 \end{array}$$

The \$2.00 coupon would be better.

- (b) If I am having breakfast with my sister and we each have a "Magnificent Seven" breakfast and we each also have coffee (\$1.89 each). How much will I save by using the 20% off coupon instead of the \$2.00 off coupon in this situation?

$$\begin{array}{r} 2(4.99) + 2(1.89) = 13.76 \\ = 9.98 + 3.78 \\ = 13.76 \end{array}$$

$$\begin{array}{r} 13.76 \\ 0.2 \\ \hline 2.752 \end{array}$$

I would save 75¢ more with the 20% off coupon.

32. Ehu buys a scooter that normally costs \$5,600. He only pays \$5,040 for the scooter.

(a) How much, in dollars, did he save?

$$\begin{array}{r} 5600 \\ - 5040 \\ \hline 560 \end{array}$$

Ehu saved \$560.

(b) What percent off the normal price does this represent?

$$5600n = 560$$
$$n = \frac{560}{5600} = \frac{1}{10} = 10\%$$

This is 10% off the normal price.

33. Anya purchased a new sofa for 22% off the original price. What percent of the original price did she pay?

$$100\% - 22\% = 78\%$$

Anya paid 78% of the original price.

34. Marian bought a blender on sale for 10% off. The original price of the blender was \$31.20. The sales tax on the purchase is 6%.

(a) How much did the 10% off save her on the blender (before tax)?

$$31.20(0.1) = 3.12$$

Marian saved \$3.12.

(b) How much did the blender cost her, before tax?

$$\begin{array}{r} 31.20 \\ - 3.12 \\ \hline 28.08 \end{array}$$

The blender cost \$28.08, before tax.

(c) She also bought a bedspread for \$50 (not on sale). If she only bought these two items, what is the subtotal of her purchase?

$$\$28.08 + 50 = \$78.08$$

Her subtotal was \$78.08.

(d) How much did her purchase cost, with tax?

$$\begin{array}{r} 78.08 \\ 0.06 \\ \hline 46848 \end{array}$$

$$\begin{array}{r} \$78.08 \\ + 4.68 \\ \hline \$82.76 \end{array}$$

The purchase cost Marian \$82.76.

35. Approximately one person out of 3000 will be struck by lightning sometime during their lifetime. What percent of people are struck by lightning sometime during their lifetime?

$$\frac{1}{3000} = \frac{1}{30} \cdot \frac{1}{100} = \frac{1}{30} \%$$

Approximately, $\frac{1}{30} \%$ of people are struck by lightning during their lifetime.

36. About 10% of all people who are struck by lightning die due to the experience. Another 70% suffer serious, permanent injury. What percent of the people who are hit by lightning live with no serious, permanent injury after being hit by lightning?

$$100\% - (10\% + 70\%) = 20\%$$

About 20% of people survive being struck by lightning with no serious permanent injury.

37. Marla budgets 20% of her monthly net income for groceries and household items. Her net income for the month is \$3000 and she spends \$750 on groceries and household items.

- (a) How much had Marla budgeted to spend on groceries and household items?

$$3000(0.2) = 600$$

Marla budgeted \$600 to spend on groceries and household items.

- (b) By how much, in dollars, did she exceed her budget?

$$750 - 600 = 150$$

Marla exceeded her budget by \$150.

- (c) By what percent did she exceed her budgeted amount for groceries and household items?

$$n(600) = 150$$

$$n = \frac{150}{600} = \frac{1}{4} = 25\%$$

She exceeded her budget for groceries and household items by 25%.

38. Harry's new job pays 140% of what his previous job's paid. He is now making \$72,280. What was his previous salary?

$$1.4n = 72,280$$

$$n = \frac{72,280}{1.4}$$

$$\begin{array}{r} 51628.571 \\ 1.4 \overline{) 72280.000} \\ \underline{-70} \\ 22 \\ \underline{14} \\ 88 \\ \underline{84} \\ 40 \\ \underline{28} \\ 120 \\ \underline{112} \\ 80 \\ \underline{70} \\ 100 \\ \underline{98} \\ 20 \end{array}$$

Harry's previous salary was \$51628.57.

39. A 37 gram granola bar contains 130 calories. The bar contains 30 calories from fat and 3 grams of fat.

- (a) Approximately what percent of the calories in the bar are from fat?

$$n(130) = 30$$

$$n = \frac{30}{130} = \frac{3}{13}$$

About $23\frac{1}{13} \%$ of the calories are fat.

$$\begin{array}{r} 0.23 \\ 13 \overline{) 3.00} \\ \underline{26} \\ 40 \\ \underline{39} \\ 1 \end{array}$$

- (b) Approximately what percent fat content is the bar by weight?

$$n(37) = 3$$

$$n = \frac{3}{37}$$

About $8\frac{4}{37} \%$ of the weight is fat.

$$\begin{array}{r} 0.08 \\ 37 \overline{) 3.00} \\ \underline{296} \\ 4 \end{array}$$

