Valid Arguments

Valid argument forms:

Law of Detachment
(*modus ponens*) $\quad p \rightarrow q$
$$\dfrac{p}{\therefore q}$$

Law of Contraposition
(*modus tollens*) $\quad p \rightarrow q$
$$\dfrac{\sim q}{\therefore \sim p}$$

Law of Syllogism $\quad p \rightarrow q$
$$\dfrac{q \rightarrow r}{\therefore p \rightarrow r}$$

Disjunctive Syllogism $\quad p \vee q$
$$\dfrac{\sim p}{\therefore q}$$

Invalid argument forms:

Fallacy of the Converse $\quad p \rightarrow q$
$$\dfrac{q}{\therefore p}$$

Fallacy of the Inverse $\quad p \rightarrow q$
$$\dfrac{\sim p}{\therefore \sim q}$$

Quote: George Pólya (1887–1985) *It is hard to have good ideas if we have little knowledge of the subject, and it is impossible if we have no knowledge. Good ideas are based on past ideas, experience, and recollections. Experience, intuition, and practice play large roles in the discovery of proofs.*

Exercises. Write a proof to verify that each of the following is a valid argument.

1. Prove the disjunctive syllogism using a previous argument form.

2. $\quad p \rightarrow q$
   $\quad \sim r \rightarrow \ \sim q$
   $$\dfrac{p}{\therefore r}$$

3. $\quad p \rightarrow (r \rightarrow q)$
   $\quad \sim q \rightarrow p$
   $$\dfrac{\sim q}{\therefore \sim r}$$

4. If Pam is prompt, then Quinn is quiet.
   If Quinn is quiet, then Sam is not shy.
   If Tim is not timid, then Sam is shy.
   Therefore, if Pam is prompt, then Tim is timid.

5. If Pam is not prompt, then Rod is reluctant.
   Pam is not prompt or Quinn is not quiet.
   If Sam is not shy, then Quinn is quiet.
   Therefore, Rod is relunctant or Sam is shy.

6. When the hour is late, I become sleepy.
   If I am sleepy, then I take a nap.
   I am not taking a nap.
   Therefore, the hour is not late.

7. Pam is prompt or Quinn is quiet.
   If Pam is prompt, then Sam is not shy.
   Neither Quinn is quiet nor Rod is relunctant.
   Therefore, Sam is not shy.