**Section 7.1 – Decimal Fractions**

 We often abbreviate how we write common fractions that have denominators that are powers of ten by writing them as decimal fractions. The same models that were used for common fractions may be used with decimal fractions: money, multibase blocks, fraction strips, area models and volume models.

 We begin by illustrating the concept of writing “abbreviated” fractions in another base with the multibase blocks model. Example:

|  |
| --- |
| *Base Three Dienes’ Multibase Blocks* |
|  | *Block* | *Flat* | *Long* | *Unit* |
| *Base Ten* | 1 |  |  |  |
| *Base ThreeTernaryFraction* | 1 | 0.1three | 0.01three | 0.001three |
| *Model with Blocks* |  |  |  |  |

Example. Consider the *ternary fraction* 0.212three, where a block represents one whole unit.

 base three language of the blocks base ten value

 0.212three 2 flats 1 long 2 units 

*Convert the following "abbreviated" fractions to a base ten common fraction (rational fraction).*

 0.321four

 0.243five

 0.532six

 0.637eight

 0.7893

***Definition.*** A *decimal fraction (decimal)* is an abbreviation for a fraction whose denominator is a power of ten, i.e., rational numbers written as decimals are fractions.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 100,000 | 10,000 | 1,000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 | 0.0001 |
| 100,000 | 10,000 | 1,000 | 100 | 10 | 1 |  |  |  |  |
| 105 | 104 | 103 | 102 | 101 | 100 | 10–1 | 10–2 | 10–3 | 10–4 |
| Hundred Thousands | Ten Thousands | Thousands | Hundreds | Tens | Ones | Tenths | Hundredths | Thousandths | Ten-thousandths |
|  |  |  |  |  |  | dime | cent | mill |  |

*Write the following in words and as decimal fractions in expanded place-value form.*

 0.24

 twenty-four hundredths

 fraction expanded form 0.24 = 

 exponential expanded form 0.24 = 2 ∙ 10–1 + 4 ∙ 10–2

 27.836

 twenty-seven and eight hundred thirty-six thousandths

 fraction expanded form 27.836 = 

 exponential expanded form 27.836 = 2 ∙ 101 + 7 ∙ 100 + 8 ∙ 10–1 + 3 ∙ 10–2 + 6 ∙ 10–3

 800.043 versus 0.843

***Note.*** When writing a decimal with no whole number part, writing a zero before the decimal point adds clarity. For example: 0.312 the 0 draws attention to the decimal point; whereas, .312 a person may overlook the decimal point.

*Express each in words and as a common fraction (rational fraction) in simplified form.*

 0.36 0.018

 thirty-six hundredths eighteen thousandths

  

*Express each common fraction (rational fraction) as a decimal fraction.*

  

  

  

  

 ***Note:*** *We will use the division algorithm to complete the above problems in the next section where we discuss the division algorithm for decimal fractions.*

You may remember that  where the 3’s repeat without end.

***Definition.*** A decimal fraction is called a *terminating decimal* if it can be represented with a finite number of nonzero digits. A decimal fraction is called a *repeating decimal* if a finite group of digits after the decimal repeat *ad infinitum*, e.g., 0.4736736736736… where 736 is the repeating group of digits, often denoted as  with the bar grouping the digits.

***Terminating Decimal Theorem***. A rational number  in simplest form can be written in terminating form if and only if the prime factorization of the denominator is of the for 2*m* ∙ 5*n*.

*Which of the following common fractions may be written as a terminating decimal?*

  

 Yes, since the prime factorization No, since the prime factorization is 38 = 2 ∙ 19.

 is 80 = 24 ∙ 5.

  

 No, since the prime Yes, since .

 factorization 7.

***Property of rational numbers.*** Every rational number may be expressed as either a terminating decimal fraction or a repeating decimal fraction.

***Definition.*** A number that is not a rational number is called an *irrational number.* Note that the decimal form of an irrational number does not terminate and does not repeat. Examples are  and .

When may a fraction be represented as a terminating decimal fraction? *We answer this question in later sections.*

*Order each list of rational numbers from the least value to the greatest value.*

 0.8, 0.123, 0.045, 0.03

 0.8 = 0.800, 0.03 = 0.030

 0.030 < 0.045 < 0.123 < 0.800

 0.03 < 0.045 < 0.123 < 0.8

 

 , , 

 0.0240 < 0.24 < 0.25 < 0.26 < 0.27

 

 ***Note.*** Students will often consider only the nonzero digits. How would you correct this error?

***Problems and Exercises***

1. *Convert the following "abbreviated" fractions to a base ten common fraction (rational fraction) or mixed number.*

 (a) 0.213four (b) 0.403five

 (c) 0.423six (d) 0.6281

 (e) 32.103four (f) 23.214five

 (g) 41.253six (h) 65.6281

2. *Convert each base ten common fraction to an “abbreviated” fraction in the designated base.*

 (a)  to base three (b)  to base four

 (c)  to base five (d)  to base six

3. (a) 4.35 + 27.8 (b) 0.00456 + 0.0357

 (c) 873 + 4.76 (d) 93.2 + 1.43 + 0.832

 (e) 456 + 45.7 + 5.38 + 70.8 (f) 59 – 3.5

 (g) 5.72 – 3.687 (h) 14.7 – 9.34

 (i) 23.83 – 26.7 (j) 0.625 – 0.25

4. (a) 54 × 7.3 (b) 8.4 × 3.6

 (c) 5.08 × 3.2 (d) 0.67 × 0.0083

 (e) 2.73 × 0.5 (f) 211.6 ÷ 0.4

 (g) 48 ÷ 1.6 (h) 7.605 ÷ 0.09

 (i) 190.4 ÷ 0.56 (j) 0.15249 ÷ 0.039

5. (a)  (b) 

 (c)  (d) 

 (e)  + 14.89 (f)  × 4.2

 (g)  (h) 

 (i) 91.2 ÷  (j) 4.8 + 9.125 + 