**Section 7.2 – Operations with Decimal Fractions**

*Addition and Subtraction of Decimal Fractions*

Why, when adding decimal fractions, are we told to line up the decimal points when using the standard addition and subtraction algorithms?

Just as with addition of whole numbers, we add the corresponding place-values.

Work the following examples to illustrate why we line up the decimal points when using the standard addition algorithm.

4.389 + 5.07

4.389 + 5.07 = 4.389 + 5.070 

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | 1 |  |  |
|  | 4 | . | 3 | 8 | 9 |
| + | 5 | . | 0 | 7 |  |
|  | 9 | . | 4 | 5 | 9 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | 1 |  |  |
|  | 4 | . | 3 | 8 | 9 |
| + | 5 | . | 0 | 7 | 0 |
|  | 9 | . | 4 | 5 | 9 |

|  |  |  |
| --- | --- | --- |
|  | = |  |
|  | = |  |
|  |  |  |

1.3 + 0.05 + 12 + 7.461

|  |  |  |
| --- | --- | --- |
| 1 | = |  |
|  | = |  |
| 12 | = | 12 |
| +7 | = | + |
|  | = |  |

1.3 + 0.05 + 12 + 7.461 = 1.300 + 0.50 + 12.000 + 7.461 = 

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 |  |  | 1 |  |  |
|  | 1 | . | 3 |  |  |
|  | 0 | . | 0 | 5 |  |
| 1 | 2 |  |  |  |  |
| + | 7 | . | 4 | 6 | 1 |
| 2 | 0 | . | 8 | 1 | 1 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 |  |  | 1 |  |  |
|  | 1 | . | 3 | 0 | 0 |
|  | 0 | . | 0 | 5 | 0 |
| 1 | 2 | . | 0 | 0 | 0 |
| + | 7 | . | 4 | 6 | 1 |
| 2 | 0 | . | 8 | 1 | 1 |

45.7 – 9.72

45.7 – 9.72 = 45.70 – 9.72 

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | 14 |  | 16 |  |
|  | 3 | 4 |  | 6 | 10 |
|  | 4 | 5 | . | 7 |  |
| – |  | 9 | . | 7 | 2 |
|  | 3 | 5 | . | 9 | 8 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | 14 |  | 16 |  |
|  | 3 | 4 |  | 6 | 10 |
|  | 4 | 5 | . | 7 | 0 |
| – |  | 9 | . | 7 | 2 |
|  | 3 | 5 | . | 9 | 8 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | = |  | = |  |
|  | = |  | = |  |
|  |  |  |  |  |

**Scientific Notation**

*If you have ever taken a Chemistry class, you may have encountered the following numbers:*

There are approximately 602,214,179,300,000,000,000,000 molecules in one mole of any substance (this quantity is called *Avagadro’s number*).

The mass of an electron is approximately 0.0000000000000000000000000009109382 grams.

Notice these two values are inconvenient to write in regular decimal notation because there are so many digits to write. When values are very large or very small it is often more manageable to write these numbers using *scientific notation.*

*Scientific notation* is a representation that uses a decimal numeral times a power of ten. The decimal value is usually written with only one nonzero digit in front of the decimal point; this form is referred to as the *normalized* form for scientific notation.

Example. The above two values written in scientific notation are:

6.022141793 × 1023 and 9.109382 × 10–28.

Note the relationship to the exponential expanded form for place-value.

***Formal Definition*:** To express a number in *scientific notation* (normalized form)*,* we write it in the form *a* × 10*k*, where *a* is a decimal with 1≤ *a* < 10 and *k* is an integer.

*Addition and Subtraction with Scientific Notation*

We use the distributive property of multiplication over addition of rational numbers with our exponent   
property *bm ∙ bn = b(m+n)*.

Example: (4.5 × 104) + (1.75 × 104)

(4.5 × 104) + (1.75 × 104) = (4.5 + 1.75) × 104

= 6.25 × 104

Example: Consider (7.5 × 103) + (5.25 × 105).

Here the powers of 10 differ, so we first need to modify the problem before applying the distributive property.

(7.5 × 103) + (5.25 × 105) = (7.5 × 103) + (5.25 × (102 × 103)) (1)

= (7.5 × 103) + ((5.25 × 102) × 103) (2)

= (7.5 × 103) + (525 × 103) (3)

= (7.5 + 525) × 103 (4)

= 532.5 × 103 (5)

= (5.325 × 102) × 103 (6)

= 5.325 × (102 × 103) (7)

= 5.325 × 105 (8)

Identify the property used for each of the above steps.

(1) Exponent property *bm ∙ bn = b(m+n)*

(2) Associative Property of Rational Number Multiplication

(3) Standard Notation from Expanded Form

(4) Distributive Property of Multiplication over Addition of Rational Numbers

(5) Closure Property for Rational Number Addition (Decimal Addition)

(6) Expanded Form

(7) Associative Property of Rational Number Multiplication

(8) Exponent property *bm ∙ bn = b(m+n)*

The same method can be used for subtraction.

*Multiplication of Decimal Fractions*

Compute 3.8 × 2.14 by the use of an algorithm of your choice. Why does the algorithm work?

|  |
| --- |
| 2.14 |
| × 3.8 |
| 1712 |
| 6420 |
| 8.132 |





Note that the counting of the number of decimal places from the right follows from the multiplication of the denominators when the decimal fractions are written as common fractions.

Also, the multiplying the two values as if they were whole numbers follow from multiplying the numerators of the common fraction form of the decimal fractions.

*Division of Decimal Fractions*

Use the standard division algorithm to compute 3.7 ÷ 0.02. Why does the algorithm work?

Method 1. We use the Compensation Property for Division to justify the moving of the decimal point when using the standard long-division algorithm to divide decimal fractions.

|  |  |
| --- | --- |
| 0 . 0 2 3 . 7 0  2. 3 7 0 .  1 8 5 .  2  1 7  1 6  1 0  1 0 | 3.7 ÷ 0.02 = (3.7 × 100) ÷ (0.02 × 100) = 370 ÷ 2 = 185  We multiply the dividend and divisor by the denominator of the common fraction form of the divisor. Since this was two place-values for the above problem, the decimal point is moved two places to the right for both the dividend and the divisor. |

Method 2. Use equivalent fractions with the relationship between division and fractions.

|  |  |
| --- | --- |
| 0 . 0 2 3 . 7 0  2. 3 7 0 .  1 8 5 .  2  1 7  1 6  1 0  1 0 | The multiple used for the equivalent fractions is determined from the place value of the divisor (denominator). |

Method 3. Use division of fractions.

|  |  |
| --- | --- |
| 0 . 0 2 3 . 7 0  2. 3 7 0 .  1 8 5 .  2  1 7  1 6  1 0  1 0 |  |

When should you stop dividing when computing the following?

1.7 ÷ 0.03 8.1 ÷ 0.7

These problems lead to topics in the next section working with nonterminating decimal fractions.

***Caution about Calculator “Rounding”***

Several methods are used for “rounding” a number to a selected decimal position. One method is called *truncation.* A decimal is said to be *truncated*, when only the front few decimal places are used. A second method is *rounding to the nearest selected place-value*. A third method is to *round up*.

Examples: Consider the repeating decimal fraction 0.666… where the selected position is the hundredths.

The rounded value using truncation would be 0.66.

The rounded value to the nearest hundredth is 0.67.

The rounded value using round up would be 0.67.

The type of rounding to be used and the position to round to should be determined by the application problem being solved.

Examples: Consider the following three problems.

A wagon has a capacity of 119 pounds. How many 12 pound bricks can be loaded into the wagon?

What is the mean amount earned by 12 students who earned a total of $121?

How many ceramic tile are needed for a 121 inch frieze if each tile measures 12 inches by 12 inches?

Calculators either round or truncate because they can only display so many decimal places on the screen. You would either need to check the instruction manual or test with a sample problem the method used by any particular calculator. Students are often not aware that the value seen on the display is not the actual value, but is a rounded approximation.

Students often want to get around the difficulty presented by repeating decimals (and very long strings of decimal digits) by using just a few decimal places by rounding the decimal. Often this gives a very close approximation for the actual arithmetic answer. But it is important to label the solution as an approximation. DO NOT USE AN EQUALS SIGN when you have rounded or truncated a decimal value in arithmetic.

Also, there are some cases in which rounding or truncating will result in a large difference in the answers to the arithmetic. For instance, what is the difference between paying 5% interest on $80,000 mortgage and paying 5.49% interest on an $80,000 mortgage over 30 years? Each of these percentage rates rounds to 5%, but the difference in what you pay is $8,737.95.

*Multiplication and Division of Values in Scientific Notation*

***Multiplication:*** To multiply a pair of numbers given in scientific notation, we use the commutative and associative properties of rational number multiplication and the exponent rule for multiplying exponential expressions that have the same base, *bm* · *bn* = *b*(*m*+*n*).

Example:Find the product (9 × 103)(5 × 102), using scientific notation. Write the solution in normalized form.

*Solution*: First, grouping the decimal parts and the powers of 10 together, we get:

(9 × 103)(5 × 102) = 9 × (103 × 5) × 102

= 9 × (5 ×103) × 102

= (9 × 5) × (103 × 102)

= 45 × 105

= (4.5 × 101) × 105

= 4.5 × (101 × 105)

= 4.5 × 106

Identify the properties for each step above.