**Section 7.3 and Section 7.4 – Nonterminating Decimals & Real Numbers**

*Express each common fraction (rational fraction) as a decimal fraction.*

  



–28

 20
 –14

 60
 –56

 40

 –35

 50

 –49

 10

 – 7

 3

  Hence, .

***Definition.*** A decimal fraction is called a *terminating decimal* if it can be represented with a finite number of nonzero digits. A decimal fraction is called a *repeating decimal* if a finite group of digits after the decimal repeat *ad infinitum*, e.g., 0.4736736736736… where 736 is the repeating group of digits, often denoted as  with the bar grouping the digits.

***Property of rational numbers.*** Every rational number may be expressed as either a terminating decimal fraction or a repeating decimal fraction.

***Note.*** When writing a decimal with no whole number part, writing a zero before the decimal point adds clarity. For example: 0.312 the 0 draws attention to the decimal point; whereas, .312 a person may overlook the decimal point.

*Express each repeating decimal as a common fraction (rational fraction) in simplified form.*

  

 We use an algebraic method where we will subtract off the nonzero repeating numerals. By multiplying by a power of ten equal to the number of digits repeating, the decimal shifts to the right. Yet all the repeating decimals positions remain in the same place-value.

 Let *N* = . Let *N* = 

 Since one position is repeating, Since two positions are repeating

 multiply by 10. multiply by 100.

|  |
| --- |
| 100*N* =  |
|  – *N* = – |
|  99*N* = 13.100 |
|   |

|  |
| --- |
| 10*N* =  |
| – *N* = – |
|  9*N* = 5.0 |
|   |

*More Examples:*  *Order the list of rational numbers from the least value to the greatest value.*

 

 Note , 0.24 = 0.240000 , 0.26 = 0.260000, 

 0.240000 < 0.250000 < 0.252525 < 0.255555 < 0.260000

 Hence, 0.24 <  <  <  < 0.26.

***Definition.*** A number whose decimal form does not terminate and does not repeat is called an *irrational number.* Note that a decimal number is not a rational number, that is, an irrational number cannot be written as a common fraction.

 Examples are , and 0.101001000100001… .

***Definition.*** The set of *real numbers* is the union of the set of rational numbers and the set of irrational numbers.

We illustrate set relationships for the sets of numbers we have defined up to now with a Venn Diagram where the universal set is the set of real numbers.

***Real***

*Natural
Numbers*

*Whole Numbers*

*Integers*

*Rational Numbers*

*Irrational Numbers*

**Mixed Decimal and Fraction Arithmetic**

*When fractions and decimals both occur in an arithmetic problem, should we work the problem using fraction or decimal arithmetic?*

 The method to use depends on the values used in the arithmetic problem.

 Example: 14.8 +  14.5 + 

 14.8 +  = 14.8 +15.75 14.5 +  = 

 = 30.55 = 

 Example: 28 ×  30 × 

 28 ×  = 28 × 1.4 30 ×  = 30 × 

 = 39.2 = 6 × 7 = 42

 Example: 5.1 +  9.1 × 

 5.1 +  = 5.1 +  9.1 ×  = 

 =  = 

Try the last example without using fractions.

 How would you multiply 9.1 ×  or 9.1 × 0.6666… to obtain an exact solution?