**Section 5.2 – Integers: Multiplication and Division**

***Motivation Problem***

*Math, Inc. had a net loss of $3,200 each day over a five day period. What was the total net loss for Math, Inc.?*

 We would consider this as a multiplication problem to obtain a total loss of $16,000 since 5(3,200) = 16,000. However, we may also consider this as a problem involving integer multiplication since a net loss of $3,200 per day could be symbolized as –3,200. Hence, we could write the problem as 5(–3,200) = –16,000 where the negative sign on the solution represents a loss of money.

***Review the Repeated Addition definition for multiplication of whole numbers.***

 Given a whole number *a* of equal sets where *a* ≠ 0, each containing *b* elements,

 we define  and 0 · *b* = 0.

 Note that multiplying two positive integers is the same as multiplying two whole numbers; therefore, we only need to consider the cases where at least one of the factors is negative. Problems occur when we extend the repeated addition definition to the integers. We will consider each of the possible cases with the three approaches: chip model, number line model, and pattern.

1. First, we consider the case of the product of a positive integer and a negative integer where, as with the multiplication of whole numbers, the first factor represents the number of sets. Consider the problem 4 ⋅ ( –3) with each of the models. Example: 4 ⋅ ( –3)

 chip model

 number-line model (measurement model)

–5

–4

–3

–2

–1

0

–6

–7

–8

–9

–10

1

–11

–12

–13

–3

–3

–3

–3

 pattern

 4 ⋅ 3 = 12

 4 ⋅ 2 = 8 In the pattern approach, we begin with the product of two positive integers

 4 ⋅ 1 = 4 and then establish a pattern as we count down in the second position

 4 ⋅ 0 = 0 to the negative integers.

 4 ⋅ (–1) = –4

 4 ⋅ (–2) = –8

 4 ⋅ (–3) = –12

 Illustrate 3 ⋅ (–2) using each of the three models.

2. A problem occurs with the model when the first factor is a negative since the first factor refers to the number of sets. *What would we mean by a negative number of sets?* One possible modification when the first factor is negative is to consider it to be referring to the opposite collection of sets. Consider the problem (–4) ⋅ 3 with each of the models. Example: (–4) ⋅ 3

 chip model

|  |  |
| --- | --- |
| 4 ⋅ 3 = 12 | (–4) ⋅ 3 = –12 |
|  |  |
| Four sets of three | The opposite of four sets of three |

 number-line model (measurement model)

7

8

9

10

11

12

6

5

4

3

2

13

1

0

–1

3

3

3

3

–12

The resulting vector of –12 is the opposite of the result of the four vectors.

 pattern

 3 ⋅ 3 = 9

 2 ⋅ 3 = 6

 1 ⋅ 3 = 3

 0 ⋅ 3 = 0

 (–1) ⋅ 3 = –3

 (–2) ⋅ 3 = –6

 (–3) ⋅ 3 = –9

 (–4) ⋅ 3 = –12

 Illustrate (–3) ⋅ 2 using each of the three models.

In both of the above two problems, the product of a negative integer and a positive integer resulted in a negative integer.

3. Finally, we consider the product of two negative factors. Consider the problem (–4) ⋅ (–3) with each of the models.

 Example. (–4) ⋅ (–3)

 chip model

|  |  |
| --- | --- |
| 4 ⋅ (–3) = –12 | (–4) ⋅ (–3) = 12 |
|  |  |
| Four sets of negative three | The opposite of four sets of negative three |

 number-line model (measurement model)

–5

–4

–3

–2

–1

0

–6

–7

–8

–9

–10

1

–11

–12

–13

–3

–3

–3

–3

+12

 pattern *The pattern approach is a much better method to illustrate the operation.*

 3 ⋅ (–3) = –9

 2 ⋅ (–3) = –6

 1 ⋅ (–3) = –3

 0 ⋅ (–3) = 0

 (–1) ⋅ (–3) = 3

 (–2) ⋅ (–3) = 6

 (–3) ⋅ (–3) = 9

 (–4) ⋅ (–3) = 12

 Illustrate (–3) ⋅ (–2) using each of the three models.

The above example illustrates that the product of two negative integers is a positive integer. These results lead to the following rules for the multiplication of integers.

***Rules for Multiplying Two Integers***

Multiply the absolute value of the integers as if they were whole numbers, then:

1. If the two integers have the same sign, the result is positive.

2. If the two integers have different signs, the result is negative.

State the property of integer multiplication.

 (a) 3 ⋅ (–5) is an integer (c) (–8) ⋅ (–5) = (–5) ⋅ (–8)

 (b) [2 ⋅ (–1)] ⋅ (–4) = 2 ⋅ [(–1) ⋅ (–4)] (d) 1 is the unique integer such that

 (–3) ⋅ 1 = 1 ⋅ (–3) = –3.

***Properties of Integer Multiplication***

1. *Closure Property for Integer Multiplication.*Let *a* and *b* be any integers. Then *ab* is a unique integer.
2. *Commutative Property of Integer Multiplication.*Let *a* and *b* be any integers. Then *ab* = *ba*.
3. *Associative Property for Integer Multiplication.*Let *a, b,* and *c* be any integers. Then *a*(*bc*) = (*ab*)*c*.
4. *Identity Property for Integer Multiplication.*Let *a* be any integer. Then *a* · 1 = 1 · *a* = *a*. The integer 1 is called the *multiplicative identity.*
5. *Distributive Property of Multiplication over Addition of Integers.*Let *a¸ b,* and *c* be any integers. Then *a*(*b* + *c*) = *ab* + *ac*.

***INTEGER DIVISION***

***Review theMissing-Factor definition for division of whole numbers.***

 Let *a* and *b* be any two whole numbers with *b* ≠ 0. Then *a* ÷ *b* = *c* if and only if *bc* = *a.*

***Definition.*** Let *a* and *b* be any two integers with *b* ≠ 0. Then *a* ÷ *b* = *c* if and only if *bc* = *a.*

Hence, the rules for division with integers are essentially the same as for multiplication of integers.

***Problems and Exercises***

1. Perform the following computations.

 (a) (–7) ⋅ (–8) (k) (–72) ÷ (–6)

 (b) 6 ⋅ (–4) (l) 15 ÷ (–3)

 (c) (–3) ⋅ 9 (m) (–54) ÷ 9

 (d) 5 ⋅ 7 (n) 63 ÷ 7

 (e) (–548) ⋅ 68 (o) (–1722) ÷ 7

 (f) (–904) ⋅ (–47) (p) (–27474) ÷ (–38)

 (g) 326 ⋅ (–305) (q) 7614 ⋅ (–47)

 (h) 525 – (–438) + (–291) (r) (–16) ⋅ [(–93) + 37]

 (i) (–9)(–5)(–4) (s) 82[(–36) – (–28)]

 (j) [32 + (–15) – 62] ÷ [(–13) – (–8)] (t) (–9)(7)(–3)(0)(–14)(–7)(4)(12)

2. For the past two days, the temperature decreased by 3° each day. What was the temperature two days ago?

3. For the past four days, the temperature increased by 5° each day. What was the temperature four days ago?

4. The temperature is expected to increase by 2° each day over the next three days. What will be the change in

 temperature?

5. The temperature is expected to decrease by 7° each day over the next four days. What will be the change in

 temperature?

6. Today, the stock price for Math, Inc. is 54. If the stock price has dropped 5 points each of the past three trading days,

 what was the stock price three days ago?

7. Kim began a card game by losing five points on each of three consecutive hands. What is Kim's score?

8. Jimmy has a hole in his pocket and lost all of his money. He lost six quarters on each of the past four days. How

 much money did Jimmy have four days ago?

9. A football team was penalized five yards on three consecutive plays. What is the team's net yardage for the three

 plays?

10. A weather balloon is losing altitude at a rate of 320 feet per hour. What is the change in the altitude over six hours?

11. A storage tank is leaking at a rate of fourteen gallons per hour. The tank currently holds eighty-five gallons. How

 much did the tank hold thirteen hours ago?