Math 303

p. 136

#1. (a)
$$\begin{array}{r} {\scriptstyle 1} \\ 981 \\ +\,421 \\ \hline 1402 \end{array}$$

(b)
$$\begin{array}{r} {\scriptstyle 1\;1} \\ 2025 \\ 1196 \\ +\,3148 \\ \hline 6369 \end{array}$$

3. (a)
$$\begin{array}{r} 863 \\ +\,752 \\ \hline 1615 \end{array}$$

(b)
$$\begin{array}{r} 368 \\ +\,257 \\ \hline 625 \end{array}$$

(Other values are possible that obtain the same sums.)

4.
$$\begin{array}{r} {\scriptstyle 1} \\ 490 \\ 120 \\ 119 \\ 185 \\ 110 \\ 0 \\ 570 \\ 250 \\ +\,185 \\ \hline 1{,}629 \end{array}$$

No, since the sum exceeds the 1500 calories his diet allows.

6.
$$\begin{array}{r} {\scriptstyle 1} \\ 3428 \\ +\,5631 \\ \hline 9059 \end{array}$$

8. (a) (i)
$$\begin{array}{r} 687 \\ +\,549 \\ \hline 16 \\ 12 \\ 11 \\ \hline 1{,}236 \end{array}$$

(ii)
$$\begin{array}{r} 359 \\ +\,673 \\ \hline 12 \\ 12 \\ 9 \\ \hline 1{,}032 \end{array}$$

[Brief answer]

(b) The sum of each place-value with exchanges is placed in its place-value position.

[Answers may vary.]

10. $16 + 31 = (1 \cdot 10 + 6) + (3 \cdot 10 + 1)$ Expanded Form

$= (1 \cdot 10 + 3 \cdot 10) + (6 + 1)$ Commutative and Associative Prop. for Whole Number Addition

$= (1 + 3)10 + (6 + 1)$ Distributive Prop. of Multiplication over Addition of Whole Numbers.

$= 4 \cdot 10 + 7$ Basic Addition Facts.

$= 47$ Expanded to Standard Form.

#11. (a) $68 + 23 = (6\cdot10+8) + (2\cdot10+3)$

$= (6\cdot10 + 2\cdot10) + (8+3)$

$= (6+2)10 + (8+3)$

$= (6+2)10 + 11$

$= (6+2)10 + 1\cdot10 + 1$

$= (6+2+1)\cdot10 + 1$

$= 9\cdot10 + 1$

$= 91$

(b) $174 + 285 = (1\cdot100 + 7\cdot10 + 4) + (2\cdot100 + 8\cdot10 + 5)$

$= (1\cdot100 + 2\cdot100) + (7\cdot10 + 8\cdot10) + (4+5)$

$= (1+2)\cdot100 + (7+8)\cdot10 + 9$

$= (1+2)\cdot100 + 15\cdot10 + 9$

$= (1+2)\cdot100 + (10+5)10 + 9$

$= (1+2)\cdot100 + 10\cdot10 + 5\cdot10 + 9$

$= (1+2)\cdot100 + 1\cdot100 + 5\cdot10 + 9$

$= (1+2+1)\cdot100 + 5\cdot10 + 9$

$= 4\cdot100 + 5\cdot10 + 9$

$= 459$

(c) $2458 + 793 = (2\cdot1000 + 4\cdot100 + 5\cdot10 + 8) + (7\cdot100 + 9\cdot10 + 3)$

$= 2\cdot1000 + (4\cdot100 + 7\cdot100) + (5\cdot10 + 9\cdot10) + (8+3)$

$= 2\cdot1000 + (4+7)\cdot100 + (5+9)\cdot10 + (1\cdot10+1)$

(Some steps have been combined here.)

$= 2\cdot1000 + (4+7)\cdot100 + (5+9+1)\cdot10 + 1$

$= 2\cdot1000 + (4+7)\cdot100 + (10+5)\cdot10 + 1$

$= 2\cdot1000 + (4+7)\cdot100 + 1\cdot100 + 5\cdot10 + 1$

$= 2\cdot1000 + (4+7+1)\cdot100 + 5\cdot10 + 1$

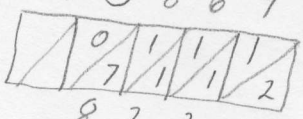
$= 2\cdot1000 + (10+2)\cdot100 + 5\cdot10 + 1$

$= 2\cdot1000 + 1\cdot1000 + 2\cdot100 + 5\cdot10 + 1$

$= (2+1)\cdot1000 + 2\cdot100 + 5\cdot10 + 1$

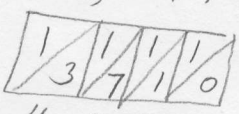
$= 3\cdot1000 + 2\cdot100 + 5\cdot10 + 1 = 3,251$

#12. (a) $4358 + 3864$



8222

(b) $4,923 + 9,897$



$14,820$

19. The numeral 5 is not a base five numeral.

22. (a)(i) $93 + 39 = 132$ $\quad 132 + 231 = 363$

(ii) $588 + 885 = 1473$ $\quad 1473 + 3741 = 5214$ $\quad 5214 + 4125 = 9339$

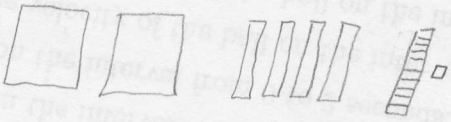
(iii) $2003 + 3002 = 5005$

(b) $79 + 97 = 176$ $\quad 176 + 671 = 847$ $\quad 847 + 748 = 1595$ $\quad 1595 + 5951 = 7546$ $\quad 7546 + 6457 = 14003$ $\quad 14003 + 30041 = 44044$

The numeral 79 took six steps.

p. 140 #3. Tira should be asked to explain why the method works. And then praised for finding a method that works. (Answers will vary.)

NAEP



(c) $251 + 14 = 237$

11

4

2

251

$75 - 50 = 25$

$2010 - 25 = 1976$

(b) 1976

p. 160 #17. (a) The exchange of ten units for one ten was lost.

(b) The student does not understand place-value, since the seven tens were placed in the hundreds position.

Math 303 - Worksheet

1. (a)
$$\begin{array}{r} 37 \\ +\,69 \\ \hline 9\,16 \\ 1\,\,0 \\ \hline 106 \end{array}$$

$$\begin{aligned} 37 &= 30+7 \\ +69 &= 60+9 \\ &\quad 90+16 \\ &= 90+10+6 \\ &= 100+6 \\ &= 106 \end{aligned}$$

$$\begin{array}{r} 37 \\ +\,69 \\ \hline 90 \\ +\,16 \\ \hline 106 \end{array} \qquad \begin{array}{r} {\scriptstyle 1}\,\,\,\,\\ 37 \\ +\,69 \\ \hline 106 \end{array} \qquad \begin{array}{r} 37 \\ +\,69 \\ \hline 106 \end{array}$$

(b)
$$\begin{array}{r} 784 \\ +\,\,46 \\ \hline 7\,12\,10 \\ 8\,\,3\,\,0 \\ \hline 830 \end{array}$$

$$\begin{aligned} 784 &= 700+80+4 \\ +46 &= \quad 40+6 \\ &\quad 700+120+10 \\ &= 700+100+20+10 \\ &= 800+30 \\ &= 830 \end{aligned}$$

$$\begin{array}{r} 784 \\ +\,\,46 \\ \hline 10 \\ 120 \\ 700 \\ \hline 830 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,1}\,\,\,\,\\ 784 \\ +\,\,46 \\ \hline 830 \end{array} \qquad \begin{array}{r} 784 \\ +\,\,46 \\ \hline 830 \end{array}$$

(d)
$$\begin{array}{r} 548 \\ +\,736 \\ \hline 12\,7\,14 \\ 1\,\,2\,\,8\,\,4 \\ \hline 1284 \end{array}$$

$$\begin{aligned} 548 &= 500+40+8 \\ +736 &= 700+30+6 \\ &\quad 1200+70+14 \\ &= 1000+200+70+10+4 \\ &= 1000+200+80+4 \\ &= 1284 \end{aligned}$$

$$\begin{array}{r} 548 \\ +\,736 \\ \hline 1200 \\ 70 \\ 14 \\ \hline 1284 \end{array} \qquad \begin{array}{r} {\scriptstyle 1}\,\,\,\,\\ 548 \\ +\,736 \\ \hline 1284 \end{array} \qquad \begin{array}{r} 548 \\ +\,736 \\ \hline 1284 \end{array}$$

(e)
$$\begin{array}{r} 8076 \\ 9704 \\ \quad 357 \\ +\,4210 \\ \hline 21\,12\,13\,17 \\ 22\,13\,14 \\ \hline 22347 \end{array}$$

$$\begin{aligned} 8076 &= 8000+\,\,0+70+6 \\ 9704 &= 9000+700+\,\,0+4 \\ 357 &= \quad\,\, 300+50+7 \\ +4210 &= 4000+200+10+0 \\ &\quad 21{,}000+1200+130+17 \\ &= 20{,}000+1000+1000+200+100+30+10+7 \\ &= 20{,}000+2000+300+40+7 \\ &= 22{,}347 \end{aligned}$$

$$\begin{array}{r} 8076 \\ 9704 \\ \quad 357 \\ +\,4210 \\ \hline 22347 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,1\,1}\,\,\,\,\\ 8076 \\ 9704 \\ \quad 357 \\ +\,4210 \\ \hline 22347 \end{array} \qquad \begin{array}{r} 8076 \\ 9704 \\ \quad 357 \\ +\,4210 \\ \hline 22347 \end{array}$$

∴

(The partial sums algorithm is on the next page.)

1. (e)

$$\begin{array}{r} 8076 \\ 9704 \\ 357 \\ +\ 4210 \\ \hline 21000 \\ 1200 \\ 130 \\ 17 \\ \hline 22347 \end{array}$$

#2. (a) base three

$$\begin{array}{r} 1201 \\ +\ 2112 \\ \hline \cancel{10}\ \cancel{10} + \cancel{10} \\ \#\ \ 2 \\ \hline 11020_{three} \end{array}$$

$$\begin{array}{r} 1201 \\ +\ 2112 \\ \hline 10 \\ 10 \\ 1000 \\ 10000 \\ \hline 11020_{three} \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\ \ 1} \\ 1201 \\ +\ 2112 \\ \hline 11020_{three} \end{array}$$

$$\begin{array}{r} 1201 \\ +\ 2112 \\ \hline 11020_{three} \end{array}$$

1	1	0	1
0	0	1	0

#2(b) base four

$$\begin{array}{r} 2301 \\ +\ 3123 \\ \hline \#\ \cancel{10}\ \cancel{2}\ \cancel{10} \\ \cancel{12}\ \ 3 \\ \hline 12030_{four} \end{array}$$

$$\begin{array}{r} 2301 \\ +\ 3123 \\ \hline 11000 \\ 1000 \\ 20 \\ 10 \\ \hline 12030_{four} \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\ \ 1} \\ 2301 \\ +\ 3123 \\ \hline 12030_{four} \end{array}$$

$$\begin{array}{r} 2301 \\ +\ 3123 \\ \hline 12030_{four} \end{array}$$

1	1	0	2	1
1	0	2	3	0

(c) base five

$$\begin{array}{r} 2403 \\ +\ 2423 \\ \hline 4\ \cancel{13}\ \cancel{2}\ \#\ \ \ \ \\ \cancel{10}\ \ \ 3 \\ \hline 10331_{five} \end{array}$$

$$\begin{array}{r} 2403 \\ +\ 2423 \\ \hline 11 \\ 20 \\ 1300 \\ 4000 \\ \hline 10331_{five} \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\ \ 1} \\ 2403 \\ +\ 2423 \\ \hline 10331_{five} \end{array}$$

$$\begin{array}{r} 2403 \\ +\ 2423 \\ \hline 10331_{five} \end{array}$$

0	1	0	1	1
4	3	2	1	
1	0	3	3	1

(d) base six

$$\begin{array}{r} 3524 \\ +\ 1453 \\ \hline 4\ \cancel{13}\ \#\ \# \\ 5\ \cancel{14}\ \cancel{12} \\ \hline 5421_{six} \end{array}$$

$$\begin{array}{r} 3524 \\ +\ 1453 \\ \hline 4000 \\ 1300 \\ 110 \\ 11 \\ \hline 5421_{six} \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 3524 \\ +\ 1453 \\ \hline 5421_{six} \end{array}$$

$$\begin{array}{r} 3524 \\ +\ 1453 \\ \hline 5421_{six} \end{array}$$

0	1	1	1	1
4	3	1	1	
5	4	2	1	

#3. (a) 3 2
+ 2 4
+0 +1
+1
1 1 1
base five

(b) 4 5 7 1
+ 2 4 5 7
6 +1 +1 +0
+2 +5
7 2 5 0
base eight

(c) 2 1 3 4
1 4 2 5
+ 2 2 2 1
5 +1 +1 +1
+0 +2 +2
1 0 2 2 4
base six

(d) 2 1 3 1
+ 4 2 3 3
+1 3 +1 4
4
1 1 4 1 4
base five

(e) 4 2 6 -
3 5 4 3
+ 2 4 5 5
+0 +2 +6 +2
11 +3 +7
1 1 3 7 2
base nine

#4. (a) Rodney is summing each column writing the result without performing any exchanges. Rodney does not understand place-value in the numeration system. He is basically writing the results using the whole-group algorithm without exchanges. The error is consistent. Use base ten blocks to show the need for exchanges.

(b) Matilda is losing the exchange. Her error is consistent. Use the base ten blocks or money to show she is losing value.

(c) Macling is performing the exchange but reversing the digits after the exchange. The base ten blocks may be used to show which value represents the greater amount in the exchange for the place-value.

(d) Tyrone is correct on all problems. He is using a variation of the partial sums algorithm.