Math 303

4/13

p. 325

#1. (a) $\{1,2,3,6,9,18\} \cap \{1,2,5,10\} = \{1,2\}$

$GCD(18,10) = 2$

$\{18,36,54,72,90,108,\ldots\} \cap \{10,20,30,40,50,60,70,80,90,\ldots$

$= \{90,180,270,\ldots\}$

$LCM(18,10) = 90$

(b) $\{1,2,3,4,6,8,12,24\} \cap \{1,2,3,4,6,9,12,18,36\} = \{1,2,3,4,6,12\}$

$GCD(24,36) = 12$

$\{24,48,72,96,120,144,\ldots\} \cap \{36,72,108,144,\ldots\}$

$= \{72,144,216,\ldots\}$

$LCM(24,36) = 72$

(c) $\{1,2,4,8\} \cap \{1,2,3,4,6,8,12,24\} \cap \{1,2,4,13,26,52\} = \{1,2,4\}$

$GCD(8,24,52) = 4$

$\{8,16,24,32,40,48,\ldots,96,\ldots,192,\ldots,288,296,304,312,\ldots\}$

$\cap \{24,48,72,96,120,144,\ldots,288,312,\ldots\}$

$\cap \{52,104,156,208,260,312,\ldots\} = \{312,624,\ldots\}$

$LCM(8,24,52) = 312$

(d) $\{1,7\} \cap \{1,3,9\} = \{1\}$

$GCD(7,9) = 1$

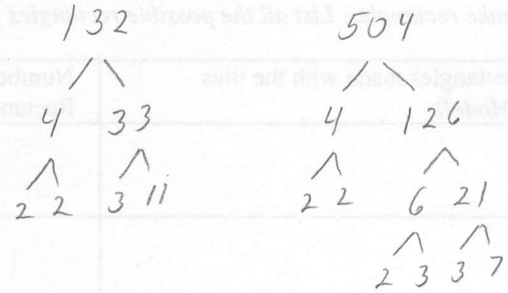
$\{7,14,21,28,35,42,49,56,63,\ldots\}$

$\cap \{9,18,27,36,45,54,63,\ldots\} = \{63,126,\ldots\}$

$LCM(7,9) = 63$

p. 325

#2. (a)



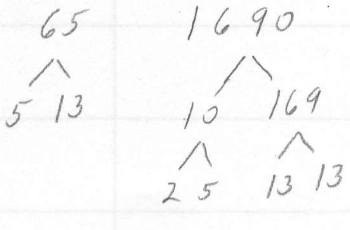
$132 = 2^2 \cdot 3 \cdot 11$

$504 = 2^3 \cdot 3^2 \cdot 7$

$GCD(132, 504) = 2^2 \cdot 3 = 12$

$LCM(132, 504) = 2^3 \cdot 3^2 \cdot 7 \cdot 11$
$= 5{,}544$

(b)



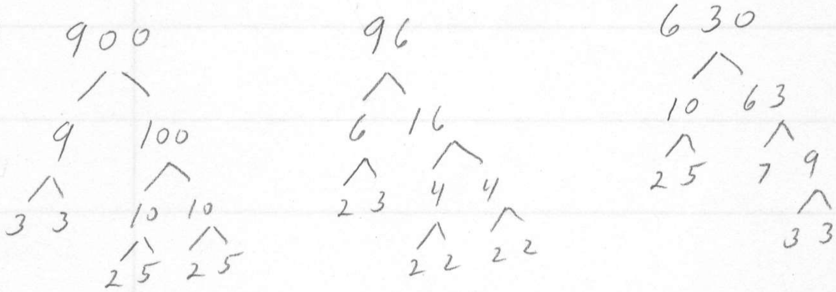
$65 = 5 \cdot 13$

$1690 = 2 \cdot 5 \cdot 13^2$

$GCD(65, 1690) = 5 \cdot 13 = 65$

$LCM(65, 1690) = 2 \cdot 5 \cdot 13^2 = 1690$

(c)



$900 = 2^2 \cdot 3^2 \cdot 5^2$

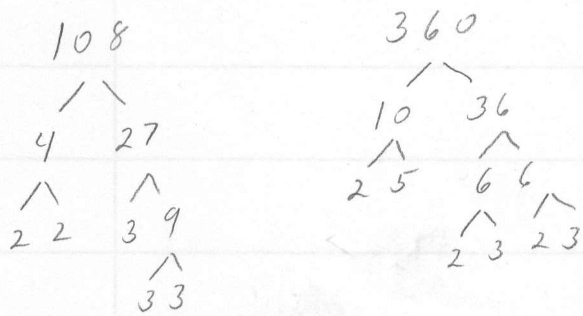
$96 = 2^5 \cdot 3$

$630 = 2 \cdot 3^2 \cdot 5 \cdot 7$

$GCD(900, 96, 630) = 2 \cdot 3 = 6$

$LCM(900, 96, 630) = 2^5 \cdot 3^2 \cdot 5^2 \cdot 7$
$= 50{,}400$

(d)



$108 = 2^2 \cdot 3^3$

$360 = 2^3 \cdot 3^2 \cdot 5$

$GCD(108, 360) = 2^2 \cdot 3^2 = 36$

$LCM(108, 360) = 2^3 \cdot 3^3 \cdot 5 = 1080$

#3. (a)

$$220\overline{)2924}^{\,13} \quad -220 \quad 724 \quad -660 \quad 64$$

$$64\overline{)220}^{\,3} \quad -192 \quad 28$$

$$28\overline{)64}^{\,2} \quad -56 \quad 8$$

$$8\overline{)28}^{\,3} \quad -24 \quad 4$$

$$4\overline{)8}^{\,2} \quad -8 \quad 0$$

$GCD(220, 2924) = 4$

(b)

$$10856\overline{)14{,}595}^{\,1} \quad -10856 \quad 3739$$

$$3739\overline{)10856}^{\,2} \quad -7478 \quad 3378$$

$$3378\overline{)3739}^{\,1} \quad -3378 \quad 361$$

$$361\overline{)3378}^{\,9} \quad -3249 \quad 129$$

$$129\overline{)361}^{\,2} \quad -258 \quad 103$$

$$103\overline{)129}^{\,1} \quad -103 \quad 26$$

$$26\overline{)103}^{\,3} \quad -78 \quad 25$$

$$25\overline{)26}^{\,1} \quad -25 \quad 1$$

$$1\overline{)25}^{\,25} \quad 25 \quad 0$$

$GCD(10856, 14595) = 1$

#4. (a) $24 = 2^3 \cdot 3$

$36 = 2^2 \cdot 3^2$

$LCM(24,36) = 2^3 \cdot 3^2 = 72$

(b) $72 = 2^3 \cdot 3^2$

$90 = 2 \cdot 3^2 \cdot 5$

$96 = 2^5 \cdot 3$

$LCM(72, 90, 96) = 2^5 \cdot 3^2 \cdot 5 = 1440$

(c) $90 = 2 \cdot 3^2 \cdot 5$

$105 = 3 \cdot 5 \cdot 7$

$315 = 3^2 \cdot 5 \cdot 7$

$LCM(90, 105, 315) = 2 \cdot 3^2 \cdot 5 \cdot 7$

$= 630$

(d) $9^{100} = 3^{200}$

$25^{100} = 5^{200}$

$LCM(9^{100}, 25^{100}) = 3^{200} \cdot 5^{200}$

$= 15^{200}$

#5. (a) $LCM(220, 2924) = 220 \cdot 2924 \div 4 = 160{,}820$

(b) $LCM(14595, 10856) = 14595 \cdot 10856 \div 1 = 158{,}443{,}320$

$$4\overline{)220}^{\,55} \quad -20 \quad 20 \quad 20$$

$$\begin{array}{r} 2924 \\ \times\ 55 \\ \hline 14620 \\ 146200 \\ \hline 160{,}820 \end{array} \qquad \begin{array}{r} 14595 \\ \times 10856 \\ \hline 87570 \\ 729750 \\ 11676000 \\ +145950000 \\ \hline 158{,}443{,}320 \end{array}$$

p. 325

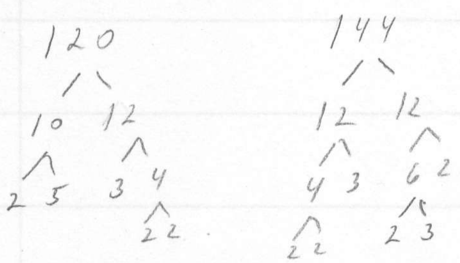
#7. (a) $15 = 3 \cdot 5$ $\quad LCM(15, 40, 60) = 2^3 \cdot 3 \cdot 5 = 120$

$40 = 2^3 \cdot 5$ $\quad$ The clocks will go off simultaneously every

$60 = 2^2 \cdot 3 \cdot 5$ $\quad$ 2 hours. The next time will be 8:00 a.m.

(b) No

#8.



$120 = 2^3 \cdot 3 \cdot 5$

$144 = 2^4 \cdot 3^2$

$GCD(120, 144) = 2^3 \cdot 3 = 24$

Midas may place at most 24 coins in each stack.

#9. $24 = 2^3 \cdot 3$ $\quad LCM(24, 45) = 2^3 \cdot 3^2 \cdot 5 = 360$

$45 = 3^2 \cdot 5$ $\quad 360 \div 24 = 15$

The fewest number of cookies José could have sold is 15.

#10. $\{12, 24, 36, \ldots\} \cap \{18, 36, \ldots\} = \{36, 72, \ldots\}$

$LCM(12, 18) = 36$

They will meet again in 36 minutes.

#11. $12 = 2^2 \cdot 3$ $\quad LCM(12, 18, 16) = 2^4 \cdot 3^2 = 144$

$18 = 2 \cdot 3^2$ $\quad$ They all pass the starting point again at the

$16 = 2^4$ $\quad$ same time in 144 minutes.

#12. (a) $LCM(a, b) = ab$ $\quad$ (b) $LCM(a, a) = GCD(a, a) = a$

(c) $GCD(a^2, a) = a$ $\quad$ (d) $GCD(a, b) = a$

$LCM(a^2, a) = a^2$ $\quad LCM(a, b) = b$

#13. (a) True, since they must be relatively prime, 2 cannot be a factor of both.

(b) True, $2 | a$ and $2 | b$ since 2 is a divisor of a and b.

(c) False, $GCD(12, 28) = 4$.

p. 325

\#15. Since 97,219,988,751 is odd, 2 and 4 are not divisors.

\#16. $\{12, 24, 36, 48, 60, \ldots\}$ $\quad LCM(12, 20) = 60$

$\{13, 26, 39, 52, 65, \ldots\}$ $\quad LCM(13, 20) = 260$

$\{20, 40, 60, \ldots\}$

The 60th caller was the first to get both the coupon and the tickets.

\#17. $\{12, 24, 36, 48, \ldots\}$ $\quad LCM(12, 16) = 48$

$\{16, 32, 48, \ldots\}$

The least amount Jackie could spend on each is \$48.

\#18. $30 = 2 \cdot 3 \cdot 5$ $\quad LCM(30, 15, 20) = 2^2 \cdot 3 \cdot 5 = 60$

$15 = 3 \cdot 5$ $\quad 60 \div 30 = 2 \quad 60 \div 15 = 4 \quad 60 \div 20 = 3$

$20 = 2^2 \cdot 5$

The least number of each that can be purchased to have an equal number of each is 2 packages of paper plates, 4 packages of paper cups and 3 packages of napkins.

\#20. $4^{10} = (2^2)^{10} = 2^{20}$

$\{1, 2, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8, 2^9, 2^{10}, 2^{11}, 2^{12}, 2^{13}, 2^{14}, 2^{15}, 2^{16}, 2^{17}, 2^{18}, 2^{19}, 2^{20}\}$

\#21. (a) $12x^4y^3 + 18x^3y^4 = 6x^3y^3(2x + 3y)$

(b) $12x^3y^2z^2 + 18x^2y^4z^3 + 24x^4y^3z^4 = (6x^2y^2z^2)(2x + 3y^2z + 4x^2yz^2)$

\#23 (a) $GCD(10!, 11!) = 10!$

$LCM(10!, 11!) = 11!$

\#24. $1{,}000{,}000{,}000 = 10^9 = (2 \cdot 5)^9 = 2^9 \cdot 5^9$

$= (512)(1{,}953{,}125)$

p. 328

#20. $xy = 1{,}000{,}000 = 10^6 = (2 \cdot 5)^6 = 2^6 \cdot 5^6 = (64)(15625)$

#22.

$$\begin{array}{r} 1037 \\ 3\overline{)3111} \\ \underline{-3} \\ 11 \\ \underline{-9} \\ 21 \\ \underline{-21} \end{array}$$

$3111 = 3(1037)$

3111 is not prime.

#23. (Answers may vary)

Example. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 = 30{,}030$

#25. $45^2 = 2025 < 2089 < 2116 = 46^2$

The greatest prime need to test is 43.