Math 303

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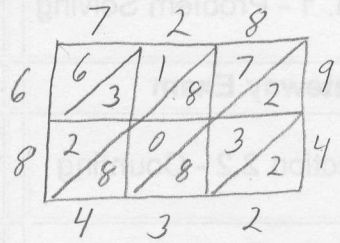
#1. (a)
$$\begin{array}{r} 426 \\ \times\ 783 \\ \hline 1278 \\ 3408 \\ 2982 \\ \hline 333558 \end{array}$$

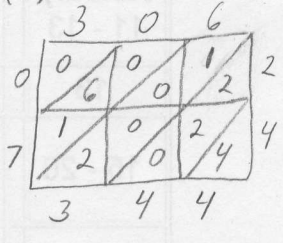
(b)
$$\begin{array}{r} 327 \\ \times\ 941 \\ \hline 327 \\ 1308 \\ 2943 \\ \hline 307107 \end{array}$$

#2. (a)



68,432

(b)



7,344

#4. (a) $5^7 \cdot 5^{12} = 5^{19}$ (b) $6^{10} \cdot 6^2 \cdot 6^3 = 6^{15}$

(c) $10^{296} \cdot 10^{17} = 10^{313}$ (d) $2^7 \cdot 10^5 \cdot 5^7 = (2^7 \cdot 5^7) \cdot 10^5 = 10^7 \cdot 10^5 = 10^{12}$

#5. (a) $2^{80} + 2^{80} = 2^{80} \cdot 1 + 2^{80} \cdot 1$ Identity Property of Multi. of Whole Numbers

$= 2^{80}(1+1)$ Distributive Prop. of Multi. over Add. of W.N.

$= 2^{80} \cdot 2 = 2^{81} < 2^{100}$

(b) $2^{101} = \underline{2 \cdot 2^{100}}$, $\underline{3 \cdot 2^{100}}$, $2^{102} = 2^2 \cdot 2^{100} = \underline{4 \cdot 2^{100}}$

Therefore, 2^{102} is the greatest.

#8.
$$\begin{array}{rll} 17 & 63 & 63 \\ 8 & \cancel{126} & 1008 \\ 4 & \cancel{252} & 1071 \\ 2 & \cancel{504} & \\ 1 & 1008 & \end{array}$$

$17 \times 63 = 1{,}071$

#9. (a)
$$\begin{array}{r} 444 \\ \times\ 3 \\ \hline 1332 \end{array}$$

There are 1,332 calories burned during three hours of cross-country skiing.

(b)
$$\begin{array}{r} 462 \\ \times\ 2 \\ \hline 924 \end{array} \qquad \begin{array}{r} 198 \\ \times\ 3 \\ \hline 594 \end{array} \qquad \begin{array}{r} {}^{8}\cancel{9}{}^{12}\cancel{2}4 \\ -\ 594 \\ \hline 330 \end{array}$$

Jane burned 330 calories more than Carolyn.

(c)
$$\begin{array}{r} 708 \\ \times\ 3 \\ \hline 2124 \end{array} \qquad \begin{array}{r} 444 \\ \times\ 5 \\ \hline 2220 \end{array} \qquad \begin{array}{r} {}^{12}{}^{10}\cancel{2}\cancel{2}\cancel{0} \\ -\ 2124 \\ \hline 96 \end{array}$$

Maurice burned 96 calories more than Lyle.

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#10. $666 \times 2 = 1332$

$1500 - 1332 = 168$

$168 \times 14 = 672 + 168 = 2352$

Glenn had a net gain of 2,352 calories, so gained less than one pound.

#18. (a) $35 \times 26 = 90$

The student worked the problem like an addition problem by only multiplying individual place-values.

Illustration:

35×26

$30 \quad (5 \times 6) \cdot 1$

$+60 \quad (3 \times 2) \cdot 10$

90

#19. (a) $56 \cdot 10 = (5 \cdot 10 + 6) \cdot 10$ Expanded Notation

$= (5 \cdot 10) \cdot 10 + 6 \cdot 10$ Distributive Prop. of Multi. over Add. of Whole numbers

$= 5 \cdot (10 \cdot 10) + 6 \cdot 10$ Associative Prop. of Multi. of Whole numbers

$= 5 \cdot 10^2 + 6 \cdot 10$ Rules of Exponents - $a^m \cdot a^n = a^{m+n}$

$= 5 \cdot 10^2 + 6 \cdot 10 + 0$ Identity Prop. for Add. of Whole Numbers.

$= 5 \cdot 10^2 + 6 \cdot 10 + 0 \cdot 1$ [Zero Prop. of Multi] or [Identity Prop. of Multi.]

$= 560$ Standard Notation

(b) $34 \cdot 10^2 = (3 \cdot 10 + 4) \cdot 10^2$ Expanded Notation

$= (3 \cdot 10) \cdot 10^2 + 4 \cdot 10^2$ Distributive Prop. of Multi over Add of Whole Numbers.

$= 3 \cdot (10 \cdot 10^2) + 4 \cdot 10^2$ Associative Prop. of Multi. of Whole Numbers

$= 3 \cdot 10^3 + 4 \cdot 10^2$ Rules of Exponents $a^m \cdot a^n = a^{m+n}$

$= 3 \cdot 10^3 + 4 \cdot 10^2 + 0 \cdot 10 + 0 \cdot 1$ Zero Property of Multiplication

$= 3400$ Standard Notation

#21. (a) $763 \times 8 = 6104$ $\quad 863 \times 7 = 6041$

763×8 is the greatest product.

(b) $678 \times 3 = 2034$ $\quad 378 \times 6 = 2268$

678×3 is the least product.

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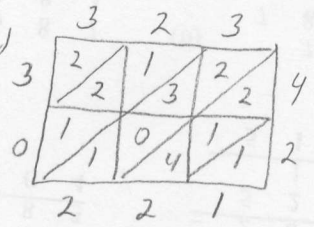
#22. (a)
$$\begin{array}{r} 213 \\ +308 \\ \hline 12 \\ 10 \\ 500 \\ \hline 522_{\text{nine}} \end{array}$$

(b)
$$\begin{array}{r} 213 \\ \times 32 \\ \hline 10 \\ 20 \\ 400 \\ 130 \\ 300 \\ 10000 \\ \hline 11300_{\text{six}} \end{array}$$

#23. (a)



$30,221_{\text{five}}$

(b) $3a + 2 = 2b + 3$

$3a = 2b + 1$

$3 \cdot 5 = 15 = 2 \cdot 7 + 1$

$a = 5$ and $b = 7$.

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#15. (a) Let's try the problem to see if it is.

$$\begin{array}{rr} 39 & 40 \\ \times 41 & \times 40 \\ \hline 39 & 1600 \\ 156 & \\ \hline 1599 & \end{array}$$

They are not the same, but 40×40 is a good estimate for 39×41.

(b) Yes, we may show it by using the distributive property.

$$\begin{aligned} 39 \cdot 41 &= 39(40+1) \\ &= 39 \cdot 40 + 39 \cdot 1 \\ &= (40-1) \cdot 40 + (40-1) \cdot 1 \\ &= 40 \cdot 40 - 1 \cdot 40 + 40 \cdot 1 - 1 \cdot 1 \\ &= 40 \cdot 40 - 40 + 40 - 1 \\ &= 40 \cdot 40 - 1 \end{aligned}$$

In general,

$$\begin{aligned} (a-1)(a+1) &= (a-1)a + (a-1) \cdot 1 \\ &= a \cdot a - 1 \cdot a + a \cdot 1 - 1 \\ &= a \cdot a - a + a - 1 \\ &= a \cdot a - 1 \end{aligned}$$

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#16. $5 + 0 = 0 + 5 = 5$

#17. (a) $ax + bx + 2x = (a+b+2)x$

(b) $3(a+b) + x(a+b) = (3+x)(a+b)$

#18.
$$\begin{array}{r} {\scriptstyle 12\,16\,10} \\ 59,\cancel{2}\cancel{8}\cancel{0} \\ -52,881 \\ \hline 6979 \end{array}$$

The total miles traveled on the trip was 6979 miles.

#19. (a) $36 \div 4 = 9$ iff $4 \times 9 = 36$

(b) $112 \div 2 = x$ iff $2x = 112$

(c) $48 \div x = 6$ iff $6x = 48$

(d) $x \div 7 = 17$ iff $7 \cdot 17 = x$

TIMSS

$$\begin{array}{r} 115 \\ \times\ 8 \\ \hline 920 \end{array}$$

Nine hundred twenty notebooks are needed for the 115 students.

$$\begin{array}{r} 41 \\ \times\ 5 \\ \hline 205 \end{array} \qquad \begin{array}{r} 51 \\ \times\ 4 \\ \hline 204 \end{array}$$

41×5 gives the largest answer.

$37 \times \square + 6 = 703 + 6$

$= 709$

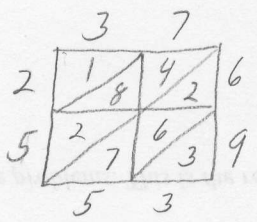
Worksheet

#1.(a)

37	69
18	~~138~~
9	276
4	~~552~~
2	~~1104~~
1	2208
	2553

$$\begin{array}{r} 37 \\ \times 69 \\ \hline 63 \\ 270 \\ 420 \\ +1800 \\ \hline 2553 \end{array}$$

$$\begin{array}{r} {\scriptstyle 4\ \ 6} \\ 37 \\ \times 69 \\ \hline 333 \\ +2220 \\ \hline 2553 \end{array}$$



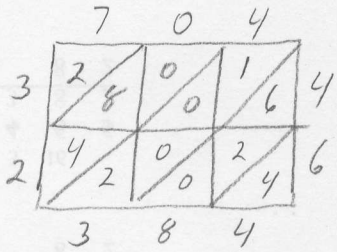
2,553

(b)

46	~~704~~
23	1408
11	2816
5	5632
2	~~11264~~
1	22528
	32,384

$$\begin{array}{r} 704 \\ \times 46 \\ \hline 24 \\ 4200 \\ 160 \\ +28000 \\ \hline 32,384 \end{array}$$

$$\begin{array}{r} 704 \\ \times 46 \\ \hline 4224 \\ +28160 \\ \hline 32,384 \end{array}$$



32,384

(c)

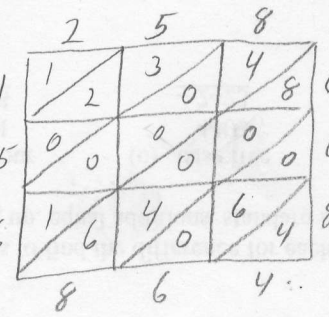
258	~~608~~
129	1216
64	~~2432~~
32	~~4864~~
16	~~9728~~
8	~~19456~~
4	~~38912~~
2	~~77824~~
1	155648
	156,864

$$\begin{array}{r} 258 \\ \times 608 \\ \hline 64 \\ 400 \\ 1600 \\ 4800 \\ 30000 \\ +120000 \\ \hline 156,864 \end{array}$$

(e)

4210	~~8076~~
2105	16152
1052	~~32304~~
526	~~64608~~
263	129216
131	258432
65	516864
32	~~1033728~~
16	~~2067456~~
8	~~4134912~~
4	~~8269824~~
2	~~16539648~~
1	33079296
	33,999,960

$$\begin{array}{r} 258 \\ \times 608 \\ \hline 2064 \\ +154800 \\ \hline 156,864 \end{array}$$

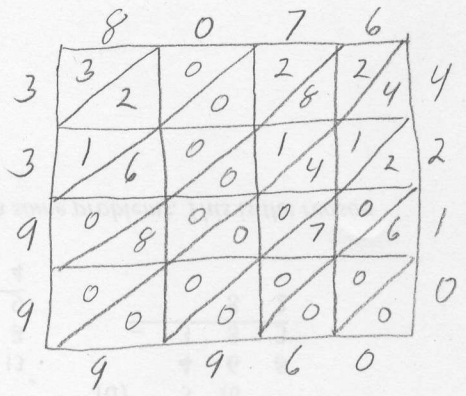


156,864

#1 (e) (continued)

$$
\begin{array}{r}
8076 \\
\times\ 4210 \\
\hline
60 \\
700 \\
80000 \\
1200 \\
14000 \\
1600000 \\
24000 \\
280000 \\
+\ 32000000 \\
\hline
33,999,960
\end{array}
$$

$$
\begin{array}{r}
8076 \\
\times\ 4210 \\
\hline
80760 \\
1615200 \\
+32304000 \\
\hline
33,999,960
\end{array}
$$

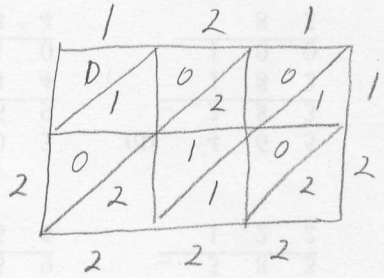


33,999,960

#2. (a) base three

$$
\begin{array}{r}
121 \\
\times\ \ 12 \\
\hline
2 \\
110 \\
200 \\
10 \\
200 \\
+\ 1000 \\
\hline
2222_{three}
\end{array}
$$

$$
\begin{array}{r}
{\scriptstyle 1} \\
121 \\
\times\ 12 \\
\hline
312 \\
+1210 \\
\hline
2222_{three}
\end{array}
$$

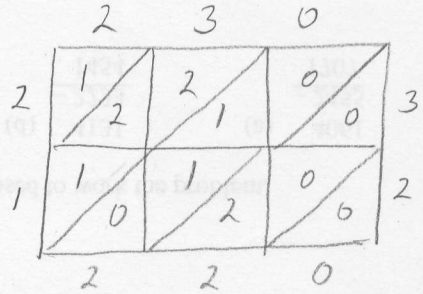


2222_{three}

(b) base four

$$
\begin{array}{r}
230 \\
\times\ 32 \\
\hline
120 \\
1000 \\
2100 \\
+12000 \\
\hline
21220_{four}
\end{array}
$$

$$
\begin{array}{r}
{\scriptstyle 2} \\
230 \\
\times\ 32 \\
\hline
1120 \\
+20100 \\
\hline
21220_{four}
\end{array}
$$

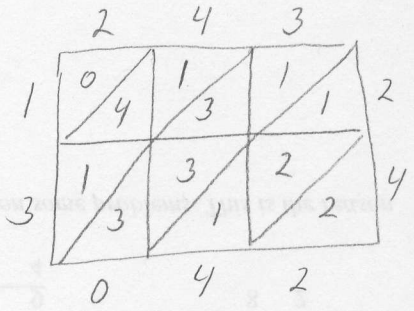


21220_{four}

#2. (c) base five

$$\begin{array}{r} 243 \\ \times\ 24 \\ \hline 22 \\ 310 \\ 1300 \\ 110 \\ 1300 \\ +4000 \\ \hline 13042_{five} \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\ \ 1} \\ {\scriptstyle 2\ \ 2} \\ 243 \\ \times\ 24 \\ \hline 2132 \\ +10410 \\ \hline 13042_{five} \end{array}$$

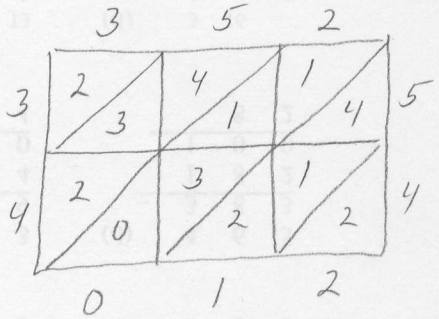


13042_{five}

(d) base six

$$\begin{array}{r} 352 \\ \times\ 54 \\ \hline 12 \\ 320 \\ 2000 \\ 140 \\ 4100 \\ +23000 \\ \hline 34012_{six} \end{array}$$

$$\begin{array}{r} {\scriptstyle 4\ \ 1} \\ {\scriptstyle 5\ \ 2} \\ 352 \\ \times\ 54 \\ \hline 2332 \\ +31240 \\ \hline 34012_{six} \end{array}$$



34012_{six}

#3. (a) $\begin{array}{r} 32 \\ \times\ 3 \\ \hline 12 \\ +210 \\ \hline 222_{four} \end{array}$

(b) $\begin{array}{r} 214 \\ \times\ 3 \\ \hline 20 \\ 30 \\ 1000 \\ \hline 1050_{six} \end{array}$

(c) $\begin{array}{r} 2134 \\ \times\ 10 \\ \hline 21340 \end{array}$

Any base greater than four.

(d) $\begin{array}{r} 142 \\ \times\ 5 \\ \hline 13 \\ 260 \\ +500 \\ \hline 1103_{seven} \end{array}$

(e) $\begin{array}{r} 421 \\ \times\ 13 \\ \hline 1463 \\ +4210 \\ \hline 5673_{eight} \end{array}$

#4. Sonda is using the procedure of an addition algorithm.

Darren's is correct.

Kareem is <u>not</u> adding the exchange value to the next term.

Kathy is adding the exchange value to the top factor before multiplying.

Ana has incorrect place-value.

Illustrate (c) for each

Sonda

$$\begin{array}{r} 82 \\ \times 37 \\ \hline 254 \end{array}$$

Kareem

$$\begin{array}{r} 82 \\ \times 37 \\ \hline 564 \\ +2460 \\ \hline 3024 \end{array}$$

Kathy

$$\begin{array}{r} 82 \\ \times 37 \\ \hline 634 \\ +2460 \\ \hline 3094 \end{array}$$

Ana

$$\begin{array}{r} 82 \\ \times 37 \\ \hline 14 \\ 6 \\ 560 \\ +240 \\ \hline 820 \end{array}$$