Math 303

p. 123 1. (Answers will vary)

Sample. How many distinct letters of the alphabet are used to write the two words "fun" and "fine".

Sol. Let $A = \{f, u, n\}$ and $B = \{f, i, n, e\}$.

$n(A) = 3$ and $n(B) = 4$ $\quad A \cup B = \{f, u, n, i, e\}$

$n(A) + n(B) = 3 + 4 \neq 5 = n(A \cup B)$.

A and B are <u>not</u> disjoint since $A \cap B = \{f, n\}$.

2. (a) True. $A \cup B = \{a, b, c, d, e\}$

$n(A) + n(B) = 3 + 2 = 5 = n(A \cup B)$

(b) False. $A \cup B = \{a, b, c\}$.

$n(A) + n(B) = 3 + 2 \neq 5 = n(A \cup B)$

(c) True. $A \cup B = \{a, b, c\}$

$n(A) + n(B) = 3 + 0 = 3 = n(A \cup B)$.

3. $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 3 + 5 - 6$

$= 2$

Two elements are common to both A and B.

4. (a) Since $n(A \cup B) = 6$ and $n(A) = 3$, $n(B)$ must be at least 3 but cannot be more than 6. Hence $n(B)$ is 3, 4, 5, or 6.

(b) Since $A \cap B = \emptyset$, $n(A) + n(B) = n(A \cup B)$.

Hence $3 + n(B) = 6$. Thus $n(B) = 3$.

5. (a) Yes, since the only possible problem is $0 + 0 = 0 \in B$

(b) Yes, since $3m + 3n = 3(m+n) \in T$ where m and n are whole numbers.

(c) Yes, the sum of two natural numbers is a natural number.

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5. (d) No, $3+5=8 \notin V$.

(e) Yes, if x and y are whole numbers greater than 10,

$x+y > 10+10 = 20 > 10$.

$x+y$ is a whole number greater than 10.

6. (a) Commutative Property of Addition for Whole Numbers

(b) Associative Property of Addition for Whole Numbers

(c) Commutative Property of Addition for Whole Numbers

(d) Identity Property of Addition for Whole Numbers

(e) Commutative Property of Addition for Whole Numbers

(f) Associative Property of Addition for Whole Numbers

7. No, since if $k=0$ we would have $a+0=b$.

Then $a=b$.

9. (a) 8, 13, 18, 23, 28, 33, 38, 43 Add 5 to the previous term.

(+5 +5 +5 +5)

(b) 98, 91, 84, 77, 70, 63, 56, 49, 42 Subtract 7 from the previous term.

(−7 −7 −7 −7 −7)

10. (a) $C=9$, since $1+8=2+7=3+6=4+5=9$.

(b) $A=8$, since $1+8=9$ but $1+9=10$. The numeral 10 has 2-digits.

(c) $C=3$, since $1+2=3$, and 1 and 2 are the digits of least value.

(d) $C=6$ or $C=8$, since $2+4=6$, $2+6=8$, $2+8=10$, $4+6=10$, etc.

(e) $B=5$, since $C=A+5$ and $A+B=C$

means $A+B=A+5$

thus $B=5$.

(f) $C=4$ or $C=8$, since $A=3B$ and $A+B=C$

means $3B+B=C$

$4B=C$

C is a multiple of 4.

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#10. (g) $C = 9$, since $A = B + 5$ and A is odd.

then $\begin{cases} A = 2+5 = 7 \\ B = 2 \end{cases}$ or $\begin{cases} A = 4+5 = 9. \\ B = 4 \end{cases}$

thus $A+B = 7+2 = 9$ or $A+B = 9+4 = 13$

we must have $7+2 = 9$.

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#2.
$$\begin{aligned} 9+7 &= 9+(1+6) && \text{Basic Fact} \\ &= (9+1)+6 && \text{Associative Prop. of Add for Whole Numbers} \\ &= 10+6 && \text{Basic Fact} \\ &= 16 && \text{Basic Fact} \end{aligned}$$