Math 303

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#5.

$$\begin{array}{r} \overset{1}{1}50 \\ 54 \\ +260 \\ \hline 464 \end{array} \qquad \begin{array}{r} \overset{2}{2}2 \\ 60 \\ 15 \\ 58 \\ {}^{2}+185 \\ \hline 340 \end{array} \qquad \begin{array}{r} 464 \\ -340 \\ \hline 124 \end{array}$$

Wally has $124 left to spend.

#7. (a)

$$\begin{array}{r} 9\overset{13}{\not 3} \\ -37 \\ 4 \\ \hline 56 \end{array}$$

(b)

$$\begin{array}{r} 3\overset{12}{\not 2}\overset{11}{\not 1} \\ -38 \\ 14 \\ \hline 283 \end{array}$$

#9. (a) Each column is added and recorded as added with no exchanges.

(b) 8+5=13. The 3 is put in the tens column and the 1 in the units.

(c) The largest is subtracted from the smallest in each place-value position. Student is following the "bad rule" of "subtract the smallest from the largest."

(d) One ten was not exchanged for ten ones.

#13. (a)

$$\begin{array}{r} 43 \\ +23 \\ \hline \end{array}$$

1/1	1/1

$1\ 2\ 1_{five}$

(b)

$$\begin{array}{r} 43 \\ -23 \\ \hline 20_{five} \end{array}$$

(c)

$$\begin{array}{r} 432 \\ +\ 23 \\ \hline \end{array}$$

0/4	1/0	1/0

$1\ 0\ 1\ 0_{five}$

(d)

$$\begin{array}{r} \overset{3}{\not 4}\overset{12}{\not 2} \\ -23 \\ \hline 14_{five} \end{array}$$

(e)

$$\begin{array}{r} 110 \\ +11 \\ \hline \end{array}$$

0/1	1/0	0/1

$1\ 0\ 0\ 1_{two}$

(f)

$$\begin{array}{r} \not 1\,\overset{1}{\overset{10}{\not 0}}\,\overset{1}{\overset{10}{\not 0}}\,\overset{10}{\not 0}\,1 \\ -\ 111 \\ \hline 1010_{two} \end{array}$$

#15. (a)

$$\begin{array}{rrr} 3\text{ hr.} & 36\text{ min.} & 58\text{ sec.} \\ +\ 5\text{ hr.} & 56\text{ min} & 27\text{ sec.} \\ \hline \not 8\text{ hr.} & \not{92}\text{ min.} & \not{85}\text{ sec} \\ & \not{93}\text{ min} & \\ \hline 9\text{ hr.} & 33\text{ min} & 25\text{ sec} \end{array}$$

(b)

$$\begin{array}{rrr} \overset{4}{\not 5}\text{ hr.} & \overset{95}{\overset{\not{35}}{\not{36}}}\text{ min} & \overset{98}{\not{38}}\text{ sec.} \\ -\ 3\text{ hr.} & 56\text{ min.} & 58\text{ sec} \\ \hline 1\text{ hr.} & 39\text{ min.} & 40\text{ sec} \end{array}$$

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#18. (a) $\begin{array}{r} {}^{3}\cancel{4}\text{ gross } {}^{15}\cancel{4}\text{ doz. } {}^{18}\cancel{6}\text{ ones} \\ -\quad\quad 5\text{ doz. } 9\text{ ones} \\ \hline 3\text{ gross } 10\text{ doz. } 9\text{ ones} \end{array}$

(b) $\begin{array}{r} 2\text{ gross } 9\text{ doz. } 7\text{ ones} \\ +\,3\text{ gross } 5\text{ doz. } 9\text{ ones} \\ \hline \cancel{5}\text{ gross } \cancel{14}\text{ doz. } \cancel{16}\text{ ones} \\ 6\text{ gross } \cancel{15}\text{ doz} \\ \hline 6\text{ gross } 3\text{ doz. } 4\text{ ones} \end{array}$

#20. (a) $\begin{array}{r} 2\,{}^{2}\underline{3}\,{}^{10}\underline{0}_{five} \\ -\quad 2\,\underline{2}_{five} \\ \hline \underline{2}\,0\,3_{five} \end{array}$

(b) $\begin{array}{r} {}^{1}2\,{}^{2}0\,{}^{2}0\,{}^{10}{}^{0}1\,{}^{10}0_{three} \\ -\quad 2\,\underline{0}\,2\,\underline{2}_{three} \\ \hline 1\,\underline{0}\,2\,\underline{1}\,1_{three} \end{array}$

#21. (a) $\begin{array}{r} 3\,{}^{3}\cancel{4}\,{}^{12}\cancel{2}3_{five} \\ -\,2\,1\,3\,2_{five} \\ \hline 1\,2\,4\,1_{five} \end{array}$

(b) $\begin{array}{r} {}^{0}\cancel{1}\,{}^{1}\cancel{0}\,{}^{1}\cancel{0}\,{}^{1}\cancel{0}\,{}^{1}\cancel{0}\,{}^{10}0_{two} \\ -\quad 1\,1\,0\,1\,1_{two} \\ \hline 1\,0\,1_{two} \end{array}$

(c) $\begin{array}{r} T\,E\,E_{twelve} \\ -\quad 1_{twelve} \\ \hline T\,E\,T_{twelve} \end{array}$

(d) $\begin{array}{r} {}^{10}\cancel{5}\,0\,0\,0_{five} \\ -\quad 1\,0\,0\,0_{five} \\ \hline 4\,0\,0\,0_{five} \end{array}$

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#6. Nice method. Cathy is using integer arithmetic.

#7. Regroup, trade, or exchange describe the actual process that is taking place; whereas, borrow implies the value will be returned and carry implies a change in value.

Subtraction Worksheet

1. (a)

```
  39
 -19
 ---
   1  (20)
  10  (30)
 + 9  (31-39)
 ---
  20
```

```
  39
 -19
 ---
  20
```

```
  39
 -19
 ---
  20
```

```
  39
 -19
 ---
  39
 +80
 ---
 x19
 + 1
 ---
  20
```

(b)

```
  284
 - 96
 ----
    4  (97-100)
   80  (110-180)
  100  (280)
 +  4  (281-284)
 ----
  188
```

```
   18 14
  2 8 4
 -  9 6
   1 10
 ------
  1 8 8
```

```
  1 17 14
  2  8  4
 -   9  6
 --------
  1  8  8
```

```
  284
 - 96
 ----
  284
 +903
 ----
 x187
 +  1
 ----
  188
```

(c)

```
  6051
 -2503
 -----
     7  (2504-2510)
    90  (2520-2600)
   400  (2700-3000)
  3000  (3000-6000)
 +  51  (6001-6051)
 -----
  3548
```

```
     10    11
  6  0  5  1
 -2  5  0  3
  3     1
 -----------
  3  5  4  8
```

```
  5 10 4 11
  6  0 5  1
 -2  5 0  3
 ----------
  3  5 4  8
```

```
  6051
 -2503
 -----
  6051
 +7496
 -----
 x3547
 +   1
 -----
  3548
```

(d)

```
  5374
 -2746
 -----
     4  (2747-2750)
    50  (2760-2800)
   200  (2900-3000)
  2000  (4000-5000)
 + 374  (5001-5374)
 -----
  2628
```

```
    13  14
  5 3 7 4
 -2 7 4 6
  3   5
 --------
  2 6 2 8
```

```
  4 13 6 14
  5  3 7  4
 -2  7 4  6
 ----------
  2  6 2  8
```

```
  5374
 -2746
 -----
  5374
 +7253
 -----
 x2627
 +   1
 -----
  2628
```

#2.

(a) base three

$$\begin{array}{r} 2201 \\ -1112 \\ \hline \end{array}$$

1 (1120)
10 (1200)
100 (2000)
+201 (2001-2201)

1012_{three}

$$\begin{array}{r} {\scriptstyle 10\ 11} \\ 22\cancel{0}X \\ -1X\,X\,2 \\ \hline 22 \end{array}$$

1012_{three}

$$\begin{array}{r} {\scriptstyle 1\ \ 2\ 10\ 11} \\ 2\cancel{2}\cancel{0}X \\ -1112 \\ \hline \end{array}$$

1012_{three}

$$\begin{array}{r} 2201 \\ -1112 \\ \hline 2201 \\ +1110 \\ \hline X1011 \\ +1 \\ \hline \end{array}$$

1012_{three}

(b) base four

$$\begin{array}{r} 3201 \\ -1123 \\ \hline \end{array}$$

1 (1130)
10 (1200)
200 (1300, 2000)
1000 (3000)
+201 (3001-3201)

2012_{four}

$$\begin{array}{r} {\scriptstyle 10\ 11} \\ 32\cancel{0}X \\ -1X\,2\,3 \\ \hline 23 \end{array}$$

2012_{four}

$$\begin{array}{r} {\scriptstyle 1\ \ 3\ 10\ 11} \\ 3\cancel{2}\cancel{0}X \\ -1123 \\ \hline \end{array}$$

2012_{four}

$$\begin{array}{r} 3201 \\ -1123 \\ \hline 3201 \\ +2210 \\ \hline X2011 \\ +1 \\ \hline \end{array}$$

2012_{four}

(c) base five

$$\begin{array}{r} 4403 \\ -2423 \\ \hline \end{array}$$

2 (2424, 2430)
20 (2440, 3000)
1000 (4000)
+403 (4001-4403)

1430_{five}

$$\begin{array}{r} {\scriptstyle 14\ 10} \\ 4\cancel{4}\cancel{0}3 \\ -2\cancel{4}23 \\ \hline 3\ 10 \end{array}$$

1430_{five}

$$\begin{array}{r} {\scriptstyle 3\ \ 13} \\ 4\cancel{4}\cancel{0}3 \\ -2423 \\ \hline \end{array}$$

1430_{five}

$$\begin{array}{r} 4403 \\ -2423 \\ \hline 4403 \\ +2021 \\ \hline X1424 \\ +1 \\ \hline \end{array}$$

1430_{five}

(d) base six

$$\begin{array}{r} 3524 \\ -1453 \\ \hline \end{array}$$

3 (1454, 1455, 1500)
100 (2000)
1000 (3000)
+524 (3001 - 3524)

2031_{six}

$$\begin{array}{r} 3\,5\overset{12}{\not 2}4 \\ -1\not 4 53 \\ \hline 5 \\ \hline 2031_{six} \end{array}$$

$$\begin{array}{r} 3\overset{4}{\not 5}\overset{12}{\not 2}4 \\ -1453 \\ \hline 2031_{six} \end{array}$$

$$\begin{array}{r} 3524 \\ -1453 \\ \hline 3524 \\ +4102 \\ \hline \not X 2030 \\ +1 \\ \hline 2031_{six} \end{array}$$

#3. (a) $\begin{array}{r} 32 \\ -24 \\ \hline 6 \end{array}$

25, 26, 27, 30, 31, 32
base eight

(b) $\begin{array}{r} 2\ 12\ 10\ 11 \\ 3\not 3\not X\not X \\ -1323 \\ \hline 1433 \end{array}$

base five

(c) $\begin{array}{r} 12\ 11\ 11 \\ 3\not 2\not X\not X \\ -\not 2\not 2\not 23 \\ \hline 333 \\ \hline 766 \end{array}$

base eight

(d) $\begin{array}{r} 3\ 10\ 12\ 11 \\ \not 4\not X\not 3\not X \\ -2233 \\ \hline 1454 \end{array}$

base six

(e) $\begin{array}{r} 4\overset{10}{\not 2}6\not X^{11} \\ -\not 2455 \\ \hline 36 \\ \hline 1707 \end{array}$

base eleven

#4. (a) Taylor is always subtracting the least value from the greatest value in each place-value position. The teacher may have made a statement such as: "Always subtract the smaller from the larger."

(b) Margarita's algorithm is a correct method. She is recording her exchanges in the fourth line and subtracting the exchanges in a second step.

(c) Scott is not doing the second exchange when the minuend contains one or more zeros. See part (c).

(d) Sally's error is similar to Scott's, except Sally is doing a double exchange when there is a zero in the minuend.