Math 303

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#1. (a) True $6(5)=30$

(b) True $30 \div 6 = 5$

(c) True (see above)

(d) True (see above)

(e) True (see above)

(f) False There is no integer x such that $30x = 6$.

#2. (a) $1+3+7+9=20$

$9 \nmid 20$

Therefore $1379 \div 9$ has a remainder, so one team would have fewer than 9 players.

(c) $2+6+1 = 9$

$9 \mid 9$

So, it is possible to assign the nine teachers the same number of the 261 students.

#3. (a) Divisible by 2.

$746,98\underline{8}$ even

Divisible by 3

$7+4+6+9+8+8=42$

$3 \mid 42$.

Divisible by 4

$4 \mid 88$

Not Divisible by 5

Last digit not 0 or 5.

Divisible by 6

Since divisible by 2 and 3.

Not Divisible by 8

$8 \nmid 988$

Not divisible by 9

$7+4+6+9+8+8=42$

$9 \nmid 42$

Not divisible by 10

Last digit is not 0.

(c) Divisible by 2

$15,81\underline{0}$ even

Divisible by 3

$1+5+8+1+0 = 15$

$3 \mid 15$

Not divisible by 4

$4 \nmid 10$

Divisible by 5

Last digit is 0.

Divisible by 6

Since divisible by 2 and 3

Not divisible by 8

Since not divisible by 4

Not divisible by 9

$1+5+8+1+0 = 15$

$9 \nmid 15$

Divisible by 10

Last digit is 0.

#3 (b) 2, since 81,34$\underline{2}$ even.

3, since $8+1+3+4+2=18$, $3 \mid 18$

not 4 and 8, since $4 \nmid 42$

6, since divisible by 2 and 3

not 5 and 10, since last digit is not 0 or 5.

9, since $8+1+3+4+2=18$ and $9 \mid 18$.

#7. (a) $7 \mid 210$ since $7(30)=210$

(b) $19 \mid (1900+38)$ since $19 \mid 1900$, $19(100)=1900$ and $19 \mid 38$, $19(2)=38$.

(c) $6 \mid 2^3 \cdot 3^2 \cdot 17^4$ since $6(2^2 \cdot 3 \cdot 17^4) = (2 \cdot 3)(2^2 \cdot 3 \cdot 17^4) = 2^3 \cdot 3^2 \cdot 17^4$.

(d) $7 \nmid (4200+22)$ since $7 \mid 4200$, $7(600)=4200$ but $7 \nmid 22$, $7(x)=22$ there is no integer x that satisfies.

#8. (a) True, (b) False, example $3 \mid 21$ but $3 \nmid 2$ and $3 \nmid 1$.

#9. (a) $3 \mid 74\underline{\ }$ $7+4+\boxed{1}=12$, $7+4+\boxed{4}=15$, $7+4+\boxed{7}=18$.

(b) $9 \mid 83\underline{\ }45$ $8+3+\boxed{7}+4+5=27$

#17. $6+8+6+8+3+9+5=45$

$3 \mid 45$, 5

Hence divisible by 3 and 5.

So 6,868,395 is divisible by 15.

#18. (a) False, $3 \mid (5+7)$ but $3 \nmid 5$.

(b) False, $3 \mid (5+7)$ but $3 \nmid 5$ and $3 \nmid 7$.

(c) False, $6 \mid (2 \cdot 3)$ but $6 \nmid 2$ and $6 \nmid 3$.

(d) True

(e) False, $5 \mid (-5)$ and $(-5) \mid 5$, but $5 \neq -5$

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#19. Let n be an integer that may be expressed as a five-digit numeral, then $n = 10000a + 1000b + 100c + 10d + e$.

Assume the sum of the digits is divisible by 9.

Then there is an integer k such that $a+b+c+d+e = 9k$.

Then

$$n = 10000a + 1000b + 100c + 10d + e$$
$$= (9999a + 999b + 99c + 9d) + (a+b+c+d+e)$$
$$= 9999a + 999b + 99c + 9d + 9k$$
$$= 9(1111a + 111b + 11c + d + k).$$

Since $1111a + 111b + 11c + d + k$ is an integer, $9 \mid n$.

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#23. (a) $3(-x) = 6$
$-3x = 6$
$x = -2$

(b) $(-2)|x| = 6$
$|x| = -3$
No integer solutions

(c) $(-x) \div 0 = -1$
No integer solutions

(d) $-(x-1) = 1-x$
$-x+1 = 1-x$
All integers

(e) $-|-x| = 5$
$|x| = -5$
No integer solutions

(f) $-x < 0$
$x > 0$
All positive integers

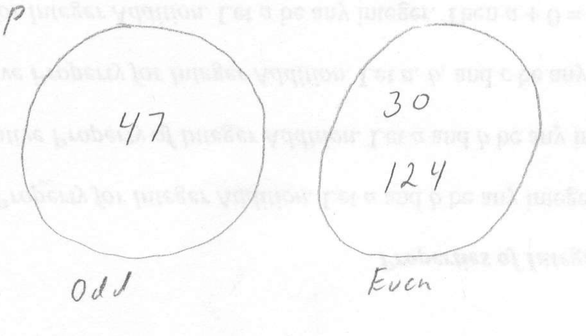
#24. (a) $3x - (1-2x)$
$= 3x - 1 + 2x$
$= 5x - 1$

(b) $(2x)^2 - 3x^2$
$= 4x^2 - 3x^2$
$= x^2$

(c) $y - x - 2(y-x)$
$= y - x - 2y + 2x$
$= -y + x = x - y$

(d) $(x-1)^2 - x^2 + 2x = x^2 - 2x + 1 - x^2 + 2x = 1$

NAEP



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#1. $2\cdot3\cdot5=30$

#2. (a) 109 prime (b) $119=7\cdot17$ composite (c) $33=3\cdot11$ composite

(d) 101 prime (e) 463 prime (f) 97 prime

(g) $2\cdot3\cdot5\cdot7+1=211$ prime (h) $2\cdot3\cdot5\cdot7-1=209=11\cdot19$ composite

#3. (a) 504

4 126

2 2 6 21

2 3 3 7

$504=2^3\cdot3^2\cdot7$

(b) 2475

25 99

5 5 9 11

3 3

$2475=3^2\cdot5^2\cdot11$

(c) 11,250

50 225

2 25 9 25

5 5 3 3 5 5

$11{,}250=2\cdot3^2\cdot5^4$

#5.
$$\begin{array}{r} 73 \\ \times73 \\ \hline 219 \\ +5110 \\ \hline 5329 \end{array}$$

$\sqrt{5669}\approx75$

73

#6. (a) $1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10$
$=2\cdot3\cdot2^2\cdot5\cdot(2\cdot3)\cdot7\cdot2^3\cdot3^2\cdot(2\cdot5)$
$=2^8\cdot3^4\cdot5^2\cdot7$

(b) $10^2\cdot26\cdot49^{10}$
$=(2\cdot5)^2\cdot(2\cdot13)\cdot(7^2)^{10}$
$=2^2\cdot5^2\cdot2\cdot13\cdot7^{20}$
$=2^3\cdot5^2\cdot7^{20}\cdot13$

(c) 251 prime (d) $1001=7\cdot143$
$=7\cdot11\cdot13$

#7. (a) 1 by 48
2 by 24
3 by 16
4 by 12

(b) One
1 by 47.

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#9. (a) $1\cdot 36 \quad \{1,2,3,4,6,9,12,18,36\}$ (b) $1\cdot 28 \quad \{1,2,4,7,14,28\}$

$2\cdot 18$ $\quad 2\cdot 14$

$3\cdot 12$ $\quad 4\cdot 7$

$4\cdot 9$

$6\cdot 6$

(c) $1\cdot 17 \quad \{1, 17\}$ (d) $144 = 2\cdot 3$

$$\begin{array}{c} 144 \\ 12 \quad 12 \\ 3\ 4 \quad 3\ 4 \\ 2\,2 \quad 2\,2 \end{array}$$

$2^0\cdot 3^0 = 1 \quad 2^0\cdot 3^1 = 3 \quad 2^0\cdot 3^2 = 9$

$2^1\cdot 3^0 = 2,\ 2^1\cdot 3^1 = 6,\ 2^1\cdot 3^2 = 18$

$2^2\cdot 3^0 = 4,\ 2^2\cdot 3^1 = 12,\ 2^2\cdot 3^2 = 36$

$2^3\cdot 3^0 = 8,\ 2^3\cdot 3 = 24,\ 2^3\cdot 3^2 = 72$

$2^4\cdot 3^0 = 16,\ 2^4\cdot 3 = 48,\ 2^4\cdot 3^2 = 144$

$\{1,2,3,4,6,8,9,12,16,18,24,32,48,72,144\}$

#16. $3^4\cdot 2^7 = (3^2\cdot 2^4)(3^2\cdot 2^3)$

$3^2\cdot 2^4$ is a factor of $3^4 2^7$.

#17. (a) $3\cdot 5\cdot 7\cdot 11\cdot 13$ (b) $(3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8) + 2 = 2(3\cdot 2\cdot 5\cdot 6\cdot 7\cdot 8 + 1)$

(c) $(3\cdot 5\cdot 7\cdot 11\cdot 13) + 5$ (d) $10! + 7 = 7(2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 8\cdot 9\cdot 10 + 1)$

$= 5(3\cdot 7\cdot 11\cdot 13 + 1)$

Each of the above have at least two distinct factors that are not 1.

#18. $2^3\cdot 3^2\cdot 25^3 = 2^3\cdot 3^2\cdot (5^2)^3$ 25 is not prime.

$= 2^3\cdot 3^2\cdot 5^6$

#19. (a) $36^{10}\cdot 49^{20}\cdot 6^{15}$

$= (2^2\cdot 3^2)^{10}\cdot (7^2)^{20}\cdot (2\cdot 3)^{15}$

$= 2^{20}\cdot 3^{20}\cdot 7^{40}\cdot 2^{15}\cdot 3^{15}$

$= 2^{35}\cdot 3^{35}\cdot 7^{40}$

(b) $100^{60}\cdot 300^{40}$

$= (2\cdot 5^2)^{60}\cdot (2\cdot 3\cdot 5^2)^{40}$

$= 2^{60}\cdot 5^{120}\cdot 2^{40}\cdot 3^{40}\cdot 5^{80}$

$= 2^{100}\cdot 3^{40}\cdot 5^{200}$

(c) $2\cdot 3^4\cdot 5^{110}\cdot 7 + 4\cdot 3^4\cdot 5^{110}$

$= 3^4\cdot 5^{110}[2\cdot 7 + 4]$

$= 3^4\cdot 5^{110}\cdot 18 = 3^4\cdot 5^{110}\cdot 2\cdot 3^2 = 2\cdot 3^6\cdot 5^{110}$

(d) $2\cdot 3\cdot 5\cdot 7\cdot 11 + 1$

$= 2311$