# Session 3 – Set Notation

***Why was there a problem answering the following question?***

> *In a game like "Who Wants to be a Millionaire" you don't hear the beginning of the question, but the end of the question is "Who is the President?" You reply, "Barack Obama." But the winning answer is "Edna Mora Szymanski."*

Just like knowing which nation or group we are referring to is important to correctly answer the question "Who is the President?", in mathematics it is important to know which group of numbers or ideas is being referred to by a particular statement or question.

In mathematics, we frequently define the group we are referring to by using the language of sets. Understanding basic set language is part of understanding how mathematics works. We will be using sets and set concepts throughout this course.

## *Some Terms from the Language of Sets*

***Set:*** A *set* is a well-defined collection of objects or ideas.

What does it mean for a set to be *well-defined*?
A set is well-defined if there is no ambiguity as to whether or not an object belongs to it, i.e., a set is defined so that we can always tell what is and what is not a member of the set.

Example: $C$ = {red, blue, yellow, green, purple} is well-defined since it is clear what is in the set.

Example: In the opening problem at the beginning of this session, the solution set to the question "Who is the President?" is *not* a well-defined collection. Does it mean only presidents of nations, or does it include presidents of companies? Of universities? Of clubs? Do we include only those serving now or all who served in the past? To be "well-defined" the collection description would have to settle all such questions.

Example: "The collection of good students at MSUM" is *not* a set. What does it mean to be "good?" Does the collection refer to past or present students? Also, does MSUM refer to Mankato, Moorhead, or some other school, college, or university? All these questions indicate the statement is ambiguous, i.e., it is not clear which students are members of this collection, hence, the collection is not well-defined.

Example: "The collection of the currently enrolled students at Minnesota State University Moorhead with a grade point average above 3.0", is a well-defined set since it is clear which students would belong to the collection.

Example: "The collection of young people that are residents of Minnesota" is *not* a set. What does it mean to be "young"? To someone that is 80, a person that is 40 is young, but a 20 year old person may not consider 40 to be young. The word "young" is an ambiguous term. Hence, the collection is not well-defined.

Example: "The collection of residents of Minnesota between the ages of 18 and 25 years old inclusive" is a well-defined set since it is fairly clear who would belong to this collection.

***Verbal, Roster, and Set-builder Notation for a Set:*** We have three ways of describing sets:

1. by name or verbal description of the elements of a set,
2. by roster (list) form by listing the elements separated by commas and using braces to enclose the list, or
3. by set-builder notation that uses a variable and a rule to describe the elements of a set.

Example: Let *S* represent the collection of states that border Minnesota (*verbal*)
*S* = {North Dakota, South Dakota, Iowa, Wisconsin, Michigan} (*roster*)
*S* = {*x* : *x* is a state bordering Minnesota} (*set-builder*)

The set-builder form is read as "*S* is the set of all *x* such that *x* is a state bordering Minnesota."

Example: Let ***N*** represent the collection of counting numbers (*verbal*)
***N*** = {1, 2, 3, 4, …} (*roster*)
***N*** = {*x* : *x* is a counting number} (*set-builder*)

Example: The set of presidents of the U.S. who were still alive June 15, 2009 (*verbal*)
{Jimmy Carter, George H. W. Bush, Bill Clinton, George W. Bush, Barack Obama} (*roster*)
{*x* : *x* was a President of the U.S. who was still alive June 15, 2009} (*set-builder*)

The set-builder form is read, "the set of all objects such that the object was a president of the U.S. who was still alive June 15, 2009."

Example: The counting number multiples of 5 that are less than 30 (*verbal*)
{5, 10, 15, 20, 25} (*roster*)
{*x* : *x* is a counting number multiple of 5 that is less than 30} (*set-builder*)

***Element of a Set*:** An object or idea in a set is called an *element* (or *member*) of the set.

Notation: The symbol ∈ is used to denote that an element is a member of a set and ∉ is used to denote that an object is not a member of a set.

Example: For set *A* = {1, 2, 3}, 1 ∈ *A*, but 12 ∉ *A*.

Example: For *S* = {*x* : *x* is a state bordering Minnesota}, Iowa ∈ *S*, but Alabama ∉ *S*.

***Empty Set*:** The *empty set* (or *null set*) is a set that has no members.

Notation: The symbol ∅ is used to represent the empty set, { }.

Example: ∅ = The collection of people attending MSUM who are 200 years old (*verbal*)
∅ = { } (*roster*)
∅ = {*x* : *x* is a person attending MSUM who is 200 years old.} (*set-builder*)

***Note***: {∅} does ***not*** symbolize the empty set; it represents a set that contains an empty set as an element and hence has a cardinality of one.

***Equal Sets***. Two sets are *equal*, if they have exactly the same elements.

Example: {a, c, t} = {c, a, t} = {t, a, c}, but {a, c, t} ≠ {a, c, t, o, r}.

Example: $\{x : x$ is a letter in the word "book"$\} = \{b, o, k\}$, but $\{b, o, k\} \neq \{b, o, t\}$.

***Note:*** The order of the elements inside the braces in the roster form does not matter.
When listing elements in the roster form, we do not repeat elements inside the set braces.

### *Sets and Whole Numbers*

In the previous session, we showed a close relationship between the concept of one-to-one correspondence and the idea of the number of elements in a set, called the cardinality of a set. (See the counting of the insects in Session 1.) Here, we formalize these relationships between sets and whole numbers.

***Sets of Numbers:*** The set of *natural numbers* (or *counting numbers*) is the set $\boldsymbol{N} = \{1, 2, 3, \ldots\}$.
The set of *whole numbers* is the set $\boldsymbol{W} = \{0, 1, 2, 3, \ldots\}$.

***Cardinal Number of a Set*:** The number of elements in a set is the *cardinal number* of that set.

Notation: If a set $A$ is equivalent to the set $\{1, 2, 3, \ldots, N\}$, we write $n(A) = N$ and say "The cardinal number of set $A$ is $N$."
Also, $n(\varnothing) = 0$. The cardinal number for an empty set is zero.

Example: Let $C = \{\#, \$, \%, \&\}$. Show $n(C) = 4$.

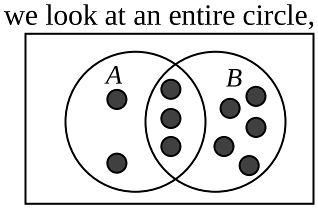
| # | $ | % | & |
|---|---|---|---|
| \| | \| | \| | \| |
| 1 | 2 | 3 | 4 |

Hence, $C$ is equivalent to $\{1, 2, 3, 4\}$ and $n(C) = 4$ since a 1-1 correspondence can be setup between $C$ and $\{1, 2, 3, 4\}$.

Example: For $M$ = {red, blue, green, yellow, orange}, $n(M) = 5$.
The symbol "$n(M) = 5$" is read, "The cardinal number of set $M$ is equal to 5."
Take the time to set up a 1-1 correspondence between $M$ and $\{1, 2, 3, 4, 5\}$.

Example: For $T = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $n(T) = 8$.
On a sheet of paper, set up a 1-1 correspondence between $T$ and $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

Example: In this picture, the circles represent sets $A$ and $B$. The dots inside are the elements of the sets.
We need to make sure we look at an entire circle, even though the circles overlap.




$n(A) = 5$ $n(B) = 8$

***Additional Notes:*** We will not give precise definitions for the terms *finite* and *infinite*. We will consider a *finite* set to be a set that has a cardinal number that is a whole number and an *infinite* set as a set that is not finite. Think of a finite set as a set that has a limited number of elements and an infinite set as a set that has an unlimited number of elements.

***Side Note.*** The cardinal number for any set equivalent to the set of all the natural numbers is $\aleph_0$, read as aleph-nought. Aleph is a letter in the Hebrew alphabet.