Session 4 – Subsets of a Set

What is the relationship between the U.S. Senate and its Judiciary Committee?

In 2009, the two senators from Minnesota, Amy Klobucher and Al Franken were both members of the U.S. Senate's Judiciary Committee, but Minnesota's 7th Congressional District representative, Collin Peterson, could not be a member of that committee.

In order to be a member of the Judiciary Committee a person must first be a member of the U.S. Senate. This is an example of a collection of objects chosen from another collection. This leads to the concept of a subset.

***Subset*:** A set A is a *subset* of a set B if every element of A is also an element of B.

Notation: $A \subseteq B$ is read, "Set A is a subset of set B."

Example: In the paragraph above, the set of members of the U.S. Senate's Judiciary Committee is a subset of the set of members of the U.S. Senate.

Example: For A = {red, blue} and B = {red, white, blue}, $A \subseteq B$ since every element of A is also an element of B.

Example: Let $C = \{a, b, c\}$ and $D = \{b, c, d, e\}$. Then $C \nsubseteq D$, read "C is not a subset of D" since a is an element of C but not an element of D. Also, $D \nsubseteq C$ since $e \in D$ and $e \notin C$.

Example: $A \subseteq A$ and $\emptyset \subseteq A$.
A set is a subset of itself since a set contains all its elements. Also, the empty set is a subset of every set, because every element in the empty set belongs to any set since the empty set has no elements.

***Listing Subsets*:** List all the subsets of $\{a, b, c\}$.

Example: The set $\{a, b, c\}$ has eight subsets. They are:
$\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, and $\{a, b, c\}$.

***Proper Subset*:** A *proper subset* is a special type of subset. There are two requirements for set A to be a proper subset of set B. They are:

1. A is a subset of B, i.e., $A \subseteq B$ ***and***
2. A is not equal to B, i.e., $A \neq B$.

Notation: $A \subset B$ is read, "Set A is a proper subset of set B."

Example: The set of members of the U.S. Senate's Judiciary Committee is a proper subset of the set of members of the U.S. Senate since not every member of the U.S. Senate is on the Judiciary Committee.

Example: For A = {red, blue} and B = {red, white, blue}, $A \subset B$ since both requirements are met:

1. $A \subseteq B$ since red and blue are in both sets A and B; ***and***
2. $A \neq B$ since set B contains the element "white" but set A does not.

***Listing Proper Subsets*:** List all the proper subsets of {*a*, *b*, *c*}.

Example: The set {*a*, *b*, *c*} has 7 proper subsets. They are:
∅, {*a*}, {*b*}, {*c*}, {*a*, *b*}, {*a*, *c*}, and {*b*, *c*}.

Note that {*a*, *b*, *c*} is not a proper subset of {*a*, *b*, *c*}. Also, note that there is always one less proper subset than there are subsets of a set since a set cannot be a proper subset of itself.

Subsets and Word Problems

We can think of a subset as being "a selection" from a specified group of objects. Think of the group of objects as a set. Then the different ways the selection can be made are the subsets.

Example: Jayne has four different color bracelets: black (b), white (w), gold (g), and silver (s). She is deciding which ones to wear today. What are her choices? How many choices does she have?

Solution: Think of the bracelets she has to choose from as a set *B* = {b, w, g, s}. Then her choices are all of the possible subsets of set *B*. Here is an organized way to list them:

0-element subsets (1)	**1- element subsets (4)**	**2- element subsets (6)**	**3-element subsets (4)**	**4-element subsets (1)**
{ }	{b}	{b, w}	{b, w, g}	{b, w, g, s}
	{w}	{b, g}	{b, w, s}	
	{g}	{b, s}	{b, g, s}	
(She decides	{s}	{w, g}	{w, g, s}	(She decides to
not to wear		{w, s}		wear all the
any bracelet.)		{g, s}		bracelets.)

Jayne has 16 choices for which bracelets to wear today. Note that $2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16$. For each choice of color, Jayne can either choose to wear the bracelet or not to wear the bracelet. This idea of multiplying the number of choices for each item to find the total number of possibilities will be covered later in the course in Session 7 with the Fundamental Counting Principle.