

Session 10 – Properties of Multiplication and the Standard Algorithm

Commutative Property of Multiplication: The *Commutative Property of Multiplication of Whole Numbers* says that the order of the factors does not change the product.

General Property: $ab = ba$

Numeric Example: $3 \times 5 = 15 = 5 \times 3$

Algebraic Example: $(3x)(4x) = (4x)(3x)$

Associative Property of Multiplication: The *Associative Property of Multiplication of Whole Numbers* says that how the factors are grouped does not change the product.

General Property: $(ab)c = a(bc)$

Numeric Example: $(2 \times 6) \times 8 = 12 \times 8$
 $= 96$
 $= 2 \times 48$
 $= 2 \times (6 \times 8)$

Algebraic Example: $2 \cdot (3x) = (2 \cdot 3)x$
 $= 6x$

Notice that in this case, regrouping allows us to simplify the expression.

Example: We show how the associative and commutative properties for multiplication of whole numbers are used to simplify an algebraic expression.

$$\begin{aligned}(3x)(4x) &= 3(x \cdot 4)x && \text{Associative Property of Multiplication} \\ &= 3(4 \cdot x)x && \text{Commutative Property of Multiplication} \\ &= (3 \cdot 4)(x \cdot x) && \text{Associative Property of Multiplication} \\ &= 12x^2\end{aligned}$$

Identity Property for Multiplication: The *Identity Property for Multiplication of Whole Numbers* says that when a value is multiplied by one the product is that value; i.e., multiplication by one does not change the value of a number. One is called the *multiplicative identity*.

General Property: $1 \cdot a = a \cdot 1 = a$

Numeric Example: $1 \cdot 5 = 5 \cdot 1 = 5$

Algebraic Example: $1(4x) = (4x) \cdot 1 = 4x$

Distributive Properties of Multiplication: The *Distributive Property of Multiplication over Addition of Whole Numbers* (the *Distributive Property of Multiplication over Subtraction of Whole Numbers*) shows us how multiplying a value times a sum (difference) may be broken into the sum (difference) of separate products.

General Property: $a(b + c) = ab + ac$ or $a(b - c) = ab - ac$

Numeric Example: $4(145) = 4(100 + 40 + 5)$
 $= 4(100) + 4(40) + 4(5)$
 $= 400 + 160 + 20$
 $= 580$

Algebraic Example: $5(3x + 9) = 5(3x) + 5(9)$
 $= (5 \cdot 3)x + 5(9)$
 $= 15x + 45$

Note that the Associative Property of Multiplication is used in the second step.

Example: The distributive property allows us to more easily perform computations mentally.

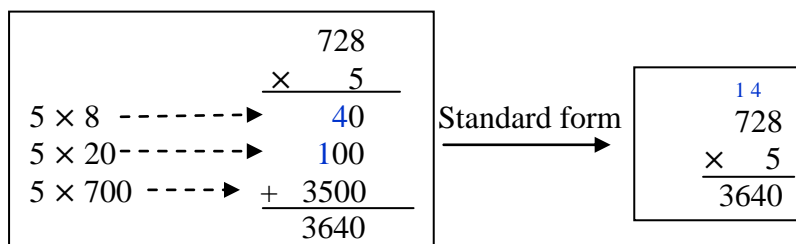
$$\begin{aligned} 7(29) &= 7(30 - 1) \\ &= 7(30) - 7(1) \\ &= 210 - 7 \\ &= 203 \end{aligned}$$

Standard Multiplication Algorithm

We can use expanded notation and the properties for whole number addition and multiplication to multiply large values. The first part of each example developing the standard algorithm shows each step in the process. Next, we write the process in a vertical form, called the partial products algorithm. Finally, we illustrate the standard multiplication algorithm, which is a short-cut method for writing partial products.

Example: Terry's take-home pay for each of the past five weeks was \$728. How much take-home pay did Terry earn over the five week period?

$5 \times 728 = 5(700 + 20 + 8)$	Expanded Notation
$= 5(8 + 20 + 700)$	Commutative and Associative Properties of Add.
$= 5(8) + 5(20) + 5(700)$	Distributive Property of Multi. over Add.
$= 40 + 100 + 3500$	
$= 3640$	Terry receive \$3640 over the five week period.



Notice that the standard form is a shortcut method for writing partial products.

Example: A manufacturer put 158 pieces of candy in each bag. How many pieces of candy would be in twenty-three bags of the candy?

$$\begin{aligned}
 23 \times 158 &= (20 + 3) \times 158 && \text{Expanded Notation} \\
 &= (3 + 20) \times 158 && \text{Commutative Property of Addition} \\
 &= 3(158) + 20(158) && \text{Distributive Property of Multi. over Add.} \\
 &= 3(100 + 50 + 8) + 20(100 + 50 + 8) && \text{Expanded Notation} \\
 &= 3(8 + 50 + 100) + 20(8 + 50 + 100) && \text{Com. and Assoc. Prop. of Add.} \\
 &= 3(8) + 3(50) + 3(100) + 20(8) + 20(50) + 20(100) && \text{Distributive Prop.} \\
 &= 24 + 150 + 300 + 160 + 1000 + 2000 \\
 &= 3634
 \end{aligned}$$

Twenty-three bags of candy would contain 3,634 pieces of candy.

We write the above computation in a vertical form (this form is called partial products) and rewrite the computation in the standard form.

$ \begin{array}{r} 158 \\ \times 23 \\ \hline 3 \times 8 \text{ -----} \rightarrow 24 \\ 3 \times 50 \text{ -----} \rightarrow 150 \\ 3 \times 100 \text{ -----} \rightarrow 300 \\ 20 \times 8 \text{ -----} \rightarrow 160 \\ 20 \times 50 \text{ -----} \rightarrow 1000 \\ 20 \times 100 \text{ -----} \rightarrow + 2000 \\ \hline 3634 \end{array} $	<p>Standard form</p> \longrightarrow	$ \begin{array}{r} 11 \\ 12 \\ 158 \\ \times 23 \\ \hline 474 \quad (3 \times 158) \\ + 3160 \quad (20 \times 158) \\ \hline 3634 \end{array} $
---	--	---