Session 14 – Like Terms and More on Solving Equations

Equations Involving More than One Operation

Adrian bought three shirts and one pair of pants for a total of $203. If Adrian paid $59 for the pants, what was the cost of each shirt?

Let x represent the cost of one shirt. Adrian would have paid $3x + 59$ for three shirts and one pair of pants. We need to solve the equation $3x + 59 = 203$ for x. This is an equation with two operations: multiplication and division. Which should we eliminate first?

Now that we know the order of operations and the inverse operations, we can solve equations like the above equation that have more than one operation. The key idea to remember is that when solving an equation, we work backwards from the order of operations used to evaluate an expression. The process is like the difference between wrapping or unwrapping a present.

Doing the arithmetic (evaluating an expression) is like wrapping the present. We take the present and put it in a box. We put the lid on the box. We wrap the box in pretty paper. We put ribbon around the box and then attach a bow.

Solving an equation is like unwrapping the present to find out what is inside. We do exactly the reverse of what was done to wrap the present. We first take off the bow, then cut the ribbon, take off the wrapping paper, remove the lid, and finally reach inside and take the present out of the box.

The order of operations tells us to do multiplication before addition or subtraction when doing arithmetic (unless otherwise indicated by parentheses). So when we solve an equation involving both multiplication and addition (or subtraction), we solve in reverse order "undoing" the addition or subtraction before we "undo" the multiplication.

Example: Solve for the cost of each shirt Adrian bought.
$$3x + 59 = 203$$
We first subtract 59 from both sides
$$(3x + 59) - 59 = 203 - 59.$$
To obtain
$$3x = 144.$$
Next, we divide both sides by 3
$$(3x) \div 3 = 144 \div 3.$$
To obtain
$$x = 48.$$

Each shirt would have cost Adrian $48.

We check the solution by substituting into the left-hand side of the equation and using the order of operations to evaluate the expression to check if we get the right-hand side.
$$3(48) + 59 = 144 + 59 = 203$$

Here is another example where we use the properties of whole numbers to justify each step in the process of solving the equation.

Step	Reason
$4x + 9 = 37$	Problem
$(4x + 9) - 9 = 37 - 9$	Subtraction Property of Equality *(See Session 8)*
$4x = 28$	Cancellation Property of Addition and Subtraction *(See Session 8)*
$\dfrac{4x}{4} = \dfrac{28}{4}$	Division Property of Equality *(See Session 11)*
$x = 7$	Cancellation Prop. of Multiplication and Division *(See Session 11)*

If you are asked to show all of the algebraic steps and give the reasons, you should show them like the example above.

If you are only asked to solve the equation, show your work as demonstrated by your instructor.

Combining Like Terms

Adrian bought three shirts and one pair of pants. After arriving at home, Adrian decided she wanted two more shirts and went back to the store to buy them. Adrian spent a total of $299. If Adrian paid $69 for the pants, what was the cost of each shirt?

We set up the equation similar to the first opening problem, but we also account for the second purchase. Let x represent the cost of one shirt. Adrian would have paid $3x + 69$ for three shirts and one pair of pants and $2x$ for the two shirts. Putting these two values together, we see that we need to solve the equation $3x + 69 + 2x = 299$ for x. This equation has two terms with the variable x. How would we combine these two terms and solve the equation?

First, we use the associative and commutative properties of addition to rewrite the equation
$$3x + 2x + 69 = 299.$$
Next, use the Distributive Property of Multiplication over Addition to rewrite the equation
$$(3 + 2)x + 69 = 299.$$
Adding 2 and 3, we obtain
$$5x + 69 = 299.$$
Finish solving the equation as before
$$(5x + 69) - 69 = 299 - 69$$
$$5x = 230$$
$$(5x) \div 5 = 230 \div 5$$
$$x = 46$$
Each shirt cost Adrian $46.

Notice that we used the distributive property to combine the two terms that had the variable x. The distributive property is the key to combining expressions with like terms.

How can we add and subtract algebraic expressions like $2x + 3x$ and $4t^2 - t^2$?

To add or subtract algebraic expressions we must have *like terms* since $2x + 3x = (2 + 3)x = 5x$ and $4t^2 - t^2 = (4 - 1)t^2 = 3t^2$. Why were these statements true? Terms are *like terms* when they have the exact same variables with the exact same exponents.

> Example: $4a$, $10a$, and a are all *like terms* because in all three expressions the variable is a and the exponent is 1. (Remember that $a = a^1$.)

> Example: $27t^2$, t^2, and $96t^2$ are all *like terms* because in all four of these expressions, the variable part is t and the exponent on t is 2.

> Example: $2a^2b$, a^2b, and $100a^2b$ are all *like terms* because in all three of these expressions, the variables are a and b, and all the a's are to the second power and all the b's are to the first power.

> Example: $10xy^2$ and $17x^2y$ are <u>NOT</u> *like terms* because, although the variables are the same, x does not have the same exponent as x^2, and the y^2 does not have the same power as y.

In the algebraic expressions we have been looking at, the number that the variable or variables are multiplied by is called the *coefficient.* For instance, in $10x^2y$ the coefficient part is the 10 and the variable part is the x^2y. For t^2, the coefficient is 1, since $1t^2 = t^2$.

Once we have *like terms*, the Distributive Property of Multiplication over Addition allows us to add or subtract the *like terms* by adding or subtracting the corresponding coefficients. Remember that the Distributive Property of Multiplication over Addition allows us to distribute multiplication over addition or subtraction. If we distribute $(4 + 3)x$, we get $4x + 3x$. (There is a notational convention that says we always put the coefficient in front of the variable). Working backwards, we can see how the distributive property lets us combine the coefficients when adding or subtracting like terms. The variable in the like term is what has been previously distributed.

Addition Example:

$$2x + 3x = (2 + 3)x$$
$$= 5x$$

Subtraction Example:

$$4t^2 - t^2 = (4 - 1)t^2$$
$$= 3t^2$$

In summary, to add or subtract like terms, we add or subtract the coefficients as indicated, keeping the variable part of the expression the same. This is an application of the Distributive Property of Multiplication over Addition (Subtraction).