Session 17 – Divisibility Tests

Computer Security

> *As mentioned in previous sessions, computer security depends on large values being difficult to factor. So, people who are trying to break encryption codes would like efficient methods for determining factors of large numbers. Such methods are only available for special cases.*

In this session, we consider some basic special cases for determining whether a value is a factor of another value. For example, when simplifying fractions we would like to quickly determine whether a value is a factor of both the numerator and the denominator. How can we quickly determine whether a number is a factor of another number?

Try this problem. Circle the numbers below that are factors of 123,456.

2 3 4 5 6 9 10

Divisibility Tests

When using factor trees to find prime factorizations of numbers it is important to be able to find factors quickly. Divisibility tests allow us to quickly find factors of numbers. Knowing divisibility tests saves us from having to try dividing by each possible factor to see whether or not it works. You probably already know a few divisibility tests. Here are the most commonly used ones.

Divisibility Tests for 2, 5, and 10

A whole number is divisible by 2 if and only if its last digit is 0, 2, 4, 6, or 8.
A whole number is divisible by 5 if and only if its last digit is 0 or 5.
A whole number is divisible by 10 if and only if its last digit is 0.

Using these tests we can see that 123,456 is divisible by 2 because its last digit is 6. We can also see that 123,456 is **not** divisible by 5 or 10 because its last digit is not 0 or 5.

Divisibility Test for 4

A whole number is divisible by 4 if and only if its last two digits are divisible by 4.

This test says that to check to see if 123,456 is divisible by 4, we only need to check if 56 is divisible by 4. Since $56 = 14 \times 4$, we know that 56 is divisible by 4; therefore, 123,456 is also divisible by 4.

Divisibility Tests for 3 and 9

A whole number is divisible by 3 if and only if the sum of its digits is divisible by 3.
A whole number is divisible by 9 if and only if the sum of its digits is divisible by 9.

To use this test to check if 123,456 is divisible by 3 or 9, we take the digits of 123,456 and find there sum. We get $1 + 2 + 3 + 4 + 5 + 6 = 21$. Since $21 = 7 \times 3$, we see that the sum of the digits is divisible by 3 and so we also know that 123,456 is divisible by 3.

We also see that since 21 is the sum of the digits, and 21 is **not** divisible by 9, we know that the number 123,456 is also **not** divisible by 9.

Divisibility Test for 6

A whole number is divisible by 6 if and only if it is divisible by both 2 and 3.

This follows directly from what we have studied about prime factorizations. We know that the prime factorization of 6 is $2 \cdot 3$. If the prime factorization of a whole number includes the primes 2 and 3, then we know that number has 6 for a factor. And if the number has 6 for a factor, it must be divisible by 6.

We have already shown that the number 123,456 is divisible by both 2 and by 3, so according to this rule the number 123,456 must also be divisible by 6.

Applying the Divisibility Tests

Circle any of the numbers below that are factors (divisors) of 34,971. Use the divisibility rules to decide. Do not divide to figure this out.

2 3 4 5 6 9 10

Solution: The last digit of 34,971 is "1". So we know that 34,971 is not divisible by 2, 5 or 10. The fact that 34,971 is not divisible by 2 also tells us that it is not divisible by either 4 or 6 (because we need a factor of 2 for either of these).

That leaves us with only 3 and 9 as possible factors (divisors) of 34,971. Both of these divisibility tests require that we look at the sum of the digits, so we sum the digits of 34,971 and get $3 + 4 + 9 + 7 + 1 = 24$. The sum of 24 is divisible by 3 but not by 9. Therefore, we conclude that 34,971 is divisible by 3 but not by 9.

This means that the only number in the list above that is a factor (divisor) of 34,971 is 3 and we should only have circled the numeral 3.