

## Session 28 – Decimal Multiplication and Division

*Solve the following problem.*

*Find the electric bill for 370 kwh's of electricity from Red River Region Co-op, which charges 0.094 dollars per kilowatt-hour with a surcharge of 0.005 dollars per kilowatt-hour and has a \$28 facility charge. Do not calculate the sales tax.*

The problem requires the multiplication of decimal fractions. We would need to multiply the total cost per kwh ( $0.094 + 0.005 = 0.099$ ) by the number of kwh's used (370) and then add the facility charge (28). Since the cost per kwh is less than a whole dollar, the multiplication is the same as multiplying with fractions.

$$0.099(370) = \frac{99}{1000} \cdot \frac{370}{1} = \frac{99(370)}{1000} = \frac{36630}{1000} = 36.630$$

Notice that we could have completed the multiplication using the standard multiplication algorithm and then moving the decimal three places from the right in the answer. That is, we do not need to multiply decimals like we did fractions; we may multiply like we did with whole numbers.

Now by adding the facility charge we obtain the final bill  $36.63 + 28 = 54.63$ . The total bill without sales tax for the 370 kwh's of electricity is \$54.63.

This problem begins the motivation for the standard algorithm for multiplying decimal fractions where we count the number of place value positions to the right of the decimal point in the factors and use that value for the number of place value positions to the right of the decimal point in the product.

### *Development of the Standard Multiplication Algorithm for Decimals*

*Why is 0.3 times 0.2 equal to 0.06, and not 0.6 like some may think?*

The rules for decimal multiplication and division follow directly from what we know about decimal place value and fraction arithmetic.

Example: Multiple 0.3(0.2).

Since  $0.3 = \frac{3}{10}$  and  $0.2 = \frac{2}{10}$ , multiplying 0.3(0.2) is the same as multiplying  $\frac{3}{10} \cdot \frac{2}{10}$ .

We know that  $\frac{3}{10} \cdot \frac{2}{10} = \frac{3 \cdot (2)}{10 \cdot (10)} = \frac{6}{100}$ , so we know that 0.3(0.2) must equal  $\frac{6}{100}$

which when written as a decimal is 0.06.

Note that to multiple 0.3(0.2) we only needed to multiple  $3(2) = 6$  and then to place the decimal point two place values from the right (0.06) since tenths time tenths equals hundredths. This motivates the process for answering the question:

### *“Why Does the General Algorithm for Multiplying Decimals Work?”*

You may remember that the general rule for multiplying decimals is to multiply the digits as if they were whole numbers and then place the decimal point in the answer based on the number of “decimal places” in the factors being multiplied.

Example: Multiply 19.6(0.073)

$$\begin{array}{r} 19.6 \leftarrow 1 \text{ decimal place (tenths)} \\ \times 0.073 \leftarrow 3 \text{ decimal places (thousandths)} \\ \hline 588 \\ + 13720 \\ \hline 1.4308 \end{array} \left. \vphantom{\begin{array}{r} 19.6 \\ \times 0.073 \\ \hline 588 \\ + 13720 \\ \hline 1.4308 \end{array}} \right\} 4 \text{ decimal places total}$$

$\leftarrow$  place the decimal point so the answer has 4 decimal places (ten-thousandths)

Note: tenths time thousandths is ten-thousandths.

Compare this to the fraction and exponent form.

$$19.6(0.073) = \frac{196}{10^1} \cdot \frac{73}{10^3} = \frac{196(73)}{10^4} = \frac{14308}{10^4} = 1.4308.$$

This rule is just a short cut description of what happens when we multiply the two decimals in their fraction forms. When a decimal value is translated into fraction form using place value, the role of the decimal point is replaced by the power of ten (place value) used in the denominator.

Examples:  $0.14 = \frac{14}{10^2}$        $1.5 = \frac{15}{10^1}$        $53.245 = \frac{53245}{10^3}$

This means that when two decimal values are multiplied, the product will be the product of these corresponding fractions.

Example:  $54.245(1.5) = \frac{54,245}{10^3} \cdot \frac{15}{10^1} = \frac{54,245 \cdot (15)}{10^3 \cdot 10^1} = \frac{813675}{(10)^4} = 81.3675$

And that in turn, means that the numerator of the product is the product of the decimal digits as whole numbers without regard to the decimal point. This explains why we can *begin* multiplying decimals by multiplying the values like we would whole numbers, without regard to the decimal point. It is because, in the fraction form, the decimal point no longer determines the place value.

***So Why Can We Just Count up the Decimal Places for the Decimal Point in the Answer?***

In the decimal form, the number of decimal places determines the place value and with it, the power of 10 for that place value.

Example: 0.3 is “three tenths” and written as a fraction is  $\frac{3}{10}$ .

One decimal place is “tenths” which means that  $10^1$  is in the denominator.

Two decimal places is “hundredths” which means that  $10^2$  is in the denominator.

When we studied simplifying exponential expressions we learned that  $b^x \cdot b^y = b^{(x+y)}$ . Applying that to our decimal multiplication situation, we can think of the denominator of the first factor is  $10^x$  and the denominator of the second factor is  $10^y$ . Then the denominator of the product must be  $10^x \cdot 10^y = 10^{(x+y)}$ . It follows that the total number of decimal places in the two factors will be the equal to the total number of decimal places in the product. So when we multiply two decimal values, we can multiply their digits as if they were whole numbers (they would be in the fraction

numerators) and the decimal point in the answer will make the same number of decimal places in the answer as the total of decimal places in the values being multiplied.

### *Development of Standard Division Algorithm for Division of Decimals*

**Why do We Move the Decimal Points when Dividing Decimals?** The rule for decimal arithmetic says that to divide by a decimal, you “move the decimal point of the divisor (outside number) to the end of the numeral and then move the decimal point of the dividend (inside number) the same number of positions (the same distance and direction).”

Example:  $0.25 \overline{)4.5}$  becomes  $25 \overline{)450}$

Once again, this is really just a short cut for what happens when we work the problem in its fraction form.

Example:  $4.5 \div 0.25$  in common fraction form is  $\frac{45}{10} \div \frac{25}{100} = \frac{45}{10} \cdot \frac{100}{25} = \frac{450}{25}$  or  $25 \overline{)450}$ .

Another way to look at the division rule is to think of the decimal division as multiplying the numerator and denominator by the same power of ten.

Example: Divide  $125.46 \div 20.5$ .

Note that this may be written as either  $20.5 \overline{)125.46}$  or  $\frac{125.46}{20.5}$ .

Eliminating the decimals in the denominator of the fraction form, we obtain

$$\frac{125.46}{20.5} = \frac{125.46(10)}{20.5(10)} = \frac{1254.6}{205}$$

So, in the standard division form, we would have

$$20.5 \overline{)125.46} \text{ or } 205 \overline{)1254.6}$$

Note that we multiplied the divisor and dividend both by the same value 10.

When completing the division be very careful with the place value in the quotient.

$$\begin{array}{r} 6.12 \\ 205 \overline{)1254.60} \\ \underline{-1230} \phantom{0} \\ 246 \phantom{0} \\ \underline{-205} \phantom{0} \\ 410 \phantom{0} \\ \underline{-410} \\ 0 \end{array}$$

We only need to get the divisor in the form of a whole number. It is easy enough to work with a decimal in the number being divided as long as we are dividing by a whole number.

However, we would still get the correct answer if we multiplied both the divisor (denominator) and dividend (numerator) by a value that would make both the divisor and the dividend a whole number. For example, in the previous problem we could have multiplied by both the divisor and dividend by 100 and then the entire problem would have become a whole number problem, but it would have exactly the same answer as before.

Example: We redo the previous example,  $125.46 \div 20.5$ .

Eliminating the decimals in the fraction form, we obtain

$$\frac{125.46}{20.5} = \frac{125.46(100)}{20.5(100)} = \frac{12546}{2050}$$

So, in the standard division form, we would have

$$20.5 \overline{)125.46} \quad \text{or} \quad 2050 \overline{)12546}$$

Note that we multiplied the divisor and dividend both by the same value 100. When completing the division be very careful with the place value in the quotient.

$$\begin{array}{r} 6.12 \\ 2050 \overline{)12546.00} \\ \underline{-12300} \phantom{00} \\ 2460 \phantom{00} \\ \underline{-2050} \phantom{00} \\ 4100 \phantom{00} \\ \underline{-4100} \phantom{00} \\ 0 \phantom{00} \end{array}$$

Notice that this second method of eliminating all the decimal points may make the divisor much larger, which often leads to a greater chance of an error. That is why it is best to use the standard algorithm of making just the divisor a whole number.

### ***Caution about Calculator “Rounding”***

Several methods are used for “rounding” a number to a selected decimal position. One method is called *truncation*. A decimal is said to be *truncated*, when only the front few decimal places are used. A second method is *rounding to the nearest selected place-value*. A third method is to *round up*.

Examples: Consider the repeating decimal fraction  $0.666\dots$  where the selected position is the hundredths.

The rounded value using truncation would be 0.66.

The rounded value to the nearest hundredth is 0.67.

The rounded value using round up would be 0.67.

The type of rounding to be used and the position to round to should be determined by the application problem being solved.

Examples: Consider the following three problems.

A wagon has a maximum capacity of 119 pounds. How many 12 pound bricks can be loaded into the wagon?

$119 \div 12 = 9.916666\dots$  Since adding too much weight could break the wagon, we would *truncate*. So, the solution is 9 bricks.

What is the mean amount earned by 12 students who earned a total of \$119?

$119 \div 12 = 9.916666\dots$  We would probably round this to the *nearest cent* since we are finding the mean. So, the solution is \$9.92.

How many ceramic tile are needed for a 121 inch frieze if each tile measures 12 inches by 12 inches?

$121 \div 12 = 10.083333\dots$  Since we would want to purchase enough tile to complete the frieze pattern, we would need to *round up*. So, the solution is 11 tiles.

Calculators either *round to the nearest* or *truncate* because they can only display so many decimal places on the screen. You would either need to read the instruction manual or test with a sample problem the method used by any particular calculator. Students are often not aware that the value seen on the display is not the actual value, but is a rounded approximation.

Students often want to get around the difficulty presented by repeating decimals (and very long strings of decimal digits) by using just a few decimal places by rounding the decimal. Often this gives a very close approximation for the actual arithmetic answer. But it is important to label the solution as an approximation. **DO NOT USE AN EQUALS SIGN when you have rounded or truncated a decimal value in arithmetic.**

Also, there are some cases in which rounding or truncating will result in a large difference in the answers to the arithmetic. For instance, what is the difference between earning 5.5% interest on \$80,000 over 30 years and earning 5.51% interest on \$80,000 over 30 years? Each of these percentage rates rounds to 5.5%, but the difference in what you receive at the end of 30 years is \$1135.34, which means the extra 0.1% is quite a bit more money.