Session 35 – Means, Frequency Tables, and Weighted Average

Interpret each of the numbers. How are they computed?

> *On July 22, 2010 the closing values of three common indices used by people for the stock market were: the Dow Jones Industrial Average was 10259.63, the NASDAQ was 2227.01, and the S&P 500 was 1086.69.*

These three measures of the value of stocks in the stock market are a weighted average (weighted mean or scaled average) of the values of sets of stocks. They are not the mean (arithmetic average) of their prices. The weighted averages consider factors other than just the price such as stock splits and stock dividends. The factors used in computing the stock averages change when companies are added or removed from the index, a stock splits, or a dividends are paid. For more information on this topic see:

<p style="text-align:center">http://en.wikipedia.org/wiki/Stock_market_index
http://en.wikipedia.org/wiki/Dow_Jones_Industrial_Average
http://en.wikipedia.org/wiki/Weighted_mean</p>

In some of the problems prior to this session, we have worked with two common types of averages that are used for measuring central tendency: mean and median. The above problem is an example of another type of average that is used where the mean and median are not a true reflection of the desired information. Often we consider some data values to be more important than other values. So, we use a weighted average (weighted mean or scaled average) to give greater value to some data over other data. Besides the stock market example given above, another situation were weighed average is used is in grading. Scores on tests are often given more value than scores on homework or quizzes.

In this session, we expand the use of the mean (arithmetic average) to large data sets and introduce weighted averages.

Mean (Arithmetic Average)

When we find the *mean* of a set of values, we are finding the *arithmetic average*. We often consider this as "evening out" the various values.

> Example: Consider five block-towers with heights of 3, 2, 4, 1, and 5 blocks. Find the mean height of these five block towers by rearranging the blocks to the same height.
>
> A child may work this problem by stacking blocks and then restacking the blocks to towers of the same size.
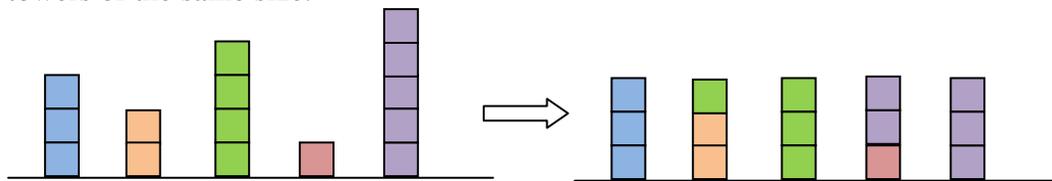
> We see from the picture that, "on the average", the 5 towers are 3 blocks tall. Notice that this is exactly the same solution as when we find the mean of the heights of the towers numerically: $\dfrac{3+2+4+1+5}{5}=\dfrac{15}{5}=3$.

Computing the Mean from a Frequency Table

For large data sets in which values repeat often, the data is usually reported in a *frequency table*. A frequency table summarizes a data set by showing how often each value occurs.

Example: Consider the following frequency table.

Data Value	10	20	30	40	50	60	70	80	90	100	110	120
Frequency	3	5	6	8	10	12	15	10	10	12	5	4

This table tells us that the data value 10 occurs 3 times in the data set, the data value 20 occurs 5 times in this data set, the data value of 30 occurs 6 times, etc. The most frequent data value is 70, which occurred 15 times.

The sum of the frequencies is 100 since $3 + 5 + 6 + 8 + \ldots + 4 = 100$, which is the total number of data values. If we compute the mean the usual way, we would have to add up all 100 separate values and then divide by 100, since there are 100 data values reported.

A quicker method is to consider the frequencies as weights for each value. So in our table, the data value 10 has a weight of 3, because it occurred 3 times, while the data value 70 has a weight of 15 because it occurred 15 times.

For example, since 10 had a frequency of 3, we can multiply $10(3) = 30$. This gives us the same total as if we had added the tens separately: $10 + 10 + 10 = 30$. For data values like 70 which have a larger number of frequencies, this save us a lot of time since $15(70)$ is more efficient to compute than adding fifteen 70's.

By extending the frequency table to include the weighted values and the totals, we compute the mean using only the table, instead of having to write out and add all 100 separate values. Notice that the sum of all of the frequencies tells us how many separate values we have.

Data Value	10	20	30	40	50	60	70	80	90	100	110	120	Totals
Frequency	3	5	6	8	10	12	15	10	10	12	5	4	100
Weighted Value	30	100	180	320	500	720	1050	800	900	1200	550	480	6,830

The mean for this data set is $\dfrac{6830}{100} = 68.3$.

Weighted Average

A *weighted average (weighted mean or scaled average)* is used when we consider some data values to be more important than other values and so we want them to contribute more to the final "average". This often occurs in the way some professors or teachers choose to assign grades in their courses. For instance, a professor may want the exam grades to "weigh" more than quiz and homework grades when computing the final grade in the course.

Example: Tully scored 70% on his midterm, 40% on his final exam, and had an average of 98% on his daily work (homework and quizzes). He believes his grade in the course will be $\dfrac{72+40+98}{3} = 70\%$. However, the course syllabus states that the midterm exam counts 40%, the final counts 50%, and all the daily work counts 10% of the final grade. What is Tully's actual grade in the course?

Solution: We need to find the grade which is composed of 40% of Tully's score on his midterm, 50% of Tully's grade on his final, and 10% of Tully's grade on his daily work.

We can make a table to help organize our computations.

	Midterm	Final Exam	Daily Grades	Totals
Component Grade	70%	40%	98%	XXXX
Component Weight	40%	50%	10%	100%
Weighted Value of Component Grades	0.40(70%) = 28%	0.50(40%) = 20%	0.10(98%) = 9.8%	57.8 points out of 100 or 57.8%

Tully's semester grade is the sum of these weighted values:
$$28\% + 20\% + 9.8\% = 57.8\%$$

This is a lower grade than Tully was expecting because his highest grade (daily) had the least weight (10%) while his lowest grade (final exam) had a much higher weight (50%).