Exercise 2.1. All triangles are isosceles triangles.

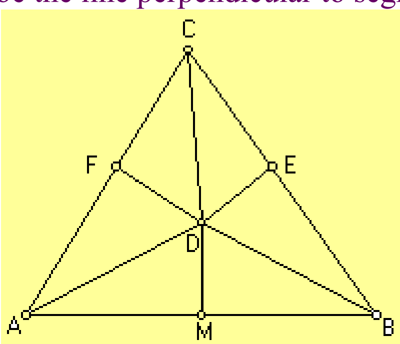
The truth of a theory is in your mind, not in your eyes.
—Albert Einstein (1879–1955)

Statement. All triangles are isosceles triangles.

(a) Identify the error/s in the proof. Is the error a logic error? A bad assumption? Explain.
(b) Identify and write a definition for each term used in the statement of the theorem and proof.
(c) Identify and state any assumptions made in the proof.
(d) Identify and state any theorems used in the proof.

(Hint. If you are unable to find the errors from reading the proof, use dynamic geometry software - such as Geometer's Sketchpad or GeoGebra - to construct the figure based on the steps in the proof.)

Proof. Given any ΔABC. Let ray r be the bisector of $\angle ACB$. Let M be the midpoint of segment AB. Let l be the line perpendicular to segment AB at M. Let D be the point of intersection of ray r and line l.



Case 1. Assume $D = M$. We have $r = \overrightarrow{CM}$ and $l = \overleftrightarrow{CM}$. Then $\angle AMC \cong \angle BMC$, since line l is perpendicular to segment AB at M. Since M is the midpoint of segment AB, $\overline{AM} \cong \overline{MB}$. Also, $\overline{CM} \cong \overline{CM}$. Hence, by SAS, $\Delta AMC \cong \Delta BMC$. Thus, $\overline{AC} \cong \overline{BC}$ and ΔABC is an isosceles triangle.

Case 2. Assume D is distinct from M. Then $\angle AMD \cong \angle BMD$, since line l is perpendicular to segment AB at M. Since M is the midpoint of segment AB, $\overline{AM} \cong \overline{MB}$. Also, $\overline{DM} \cong \overline{DM}$. Hence, by SAS, $\Delta AMD \cong \Delta BMD$. Thus $\overline{AD} \cong \overline{BD}$. Let E be the foot of the perpendicular line from D to line BC. Let F be the foot of the perpendicular line from D to line AC. Then $\angle CFD$ and $\angle CED$ are right angles. Thus $\angle CFD \cong \angle CED$. Since ray r bisects $\angle ACB$, $\angle FCD \cong \angle ECD$. Also, $\overline{CD} \cong \overline{CD}$. Hence, by AAS, $\Delta FCD \cong \Delta ECD$. Thus $\overline{FD} \cong \overline{ED}$ and $\overline{FC} \cong \overline{EC}$. Since $\overline{AD} \cong \overline{BD}$, $\overline{FD} \cong \overline{ED}$, and $\angle AFD$ and $\angle BED$ are right angles, by HL, $\Delta AFD \cong \Delta BED$. Hence, $\overline{AF} \cong \overline{BE}$. Since $\overline{AF} \cong \overline{BE}$ and $\overline{FC} \cong \overline{EC}$, we have that

$$AC = AF + FC = BE + EC = BC.$$

Hence, $\overline{AC} \cong \overline{BC}$ and ΔABC is an isosceles triangle.
Therefore, since the triangle was arbitrarily chosen, all triangles are isosceles.//