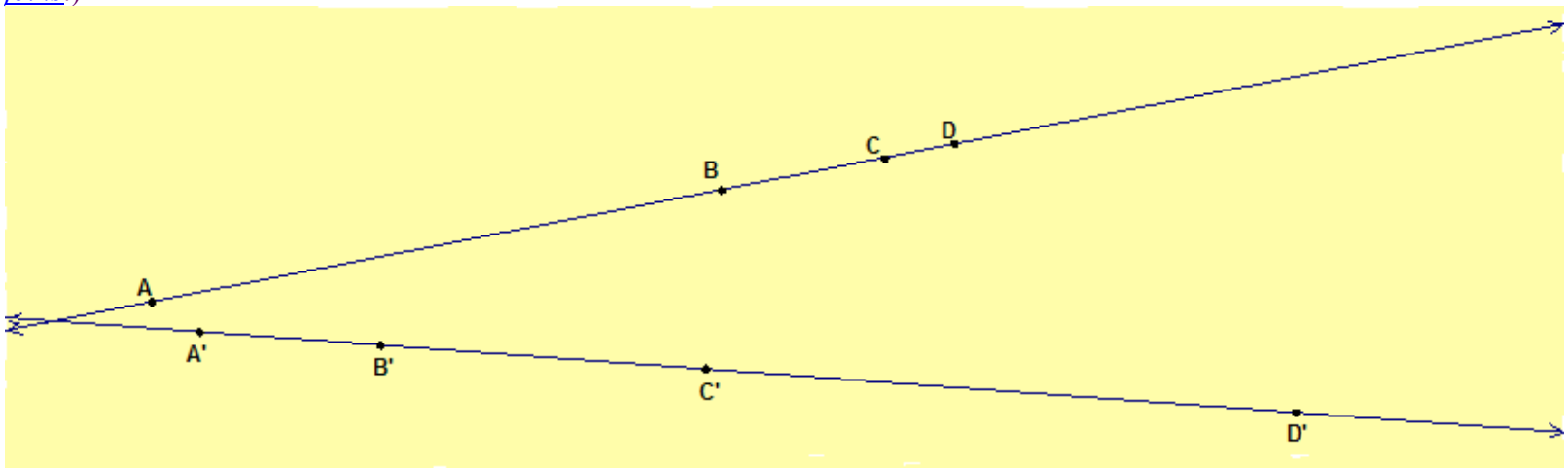
Exercise 4.38. Investigation for an Alternate Construction of a Projectivity

Art, like morality, consists of drawing the line somewhere.
— G. K. Chesterton (1874–1936)

The construction of the proof of Theorem 4.10 requires several steps to construct a projectivity between two pencils of three points. Another method is often used that is more efficient. In the given diagram, the two pencils of points are *projectively related*, $ABCD \wedge A'B'C'D'$. Draw the lines and consider the points $AB' \cdot BA'$, $AC' \cdot CA'$, $AD' \cdot DA'$, $BC' \cdot CB'$, $BD' \cdot DB'$, and $CD' \cdot DC'$. (The pairs of lines, you drew, are called *cross joins*.)



(a) How are the points related to each other?

(b) State your result as a conjecture.

(c) Now let E be an arbitrary point on the pencil of points with A, B, C, D. Based on your conjecture, construct the image of E.

Explore further with a dynamic interactive diagram GeoGebra or JavaSketchpad. Revise your conjecture, if needed.
You may also explore using the prepared Geometer's Sketchpad sketch found in the Appendix - Sketchpad Sketches.

4.6.3 Harmonic Sets and Projectivity 4.6.4 Alternate Construction of a Projectivity

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