4.2.2 Basic Theorems

New ideas pass through three periods: It can't be done. It probably can be done, but it's not worth doing. I knew it was a good idea all along!
— *Arthur C. Clarke (1917–2008)*

A projective geometry is a non-Euclidean geometry since an immediate result of Axiom 2 is that there are no parallel lines. *(The proofs for Theorems 4.1–4.4 are on the chapters solution page.)*

Theorem 4.1. *(Dual of Axiom 1)* **Any two distinct lines are incident with exactly one point.**

Exercise 4.1. Write the proof for Theorem 4.1.

Theorem 4.2. **There exist a point and a line that are not incident.**

Exercise 4.2. Prove Theorem 4.2. *(Caution: You may not assume the existence of any points or any lines.)*

Theorem 4.3. **Every line is incident with at least three distinct points.**

Exercise 4.3. Prove Theorem 4.3. *(Caution: You may not assume a line has any points.)*

Theorem 4.4. **Every line is incident with at least four distinct points.**

Exercise 4.4. Prove Theorem 4.4. *(Hint: You may need to use Axiom 4.)*

Exercise 4.5. Prove the existence of a complete quadrangle.

Exercise 4.6. Is it possible to extend Theorems 4.3 and 4.4 to an arbitrary number of points? Explain.

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