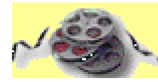
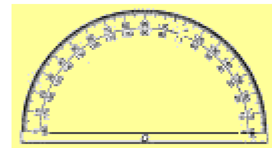
2.4.2 Angles and Angle Measure

The right angle from which to approach any problem is the try angle.



Definitions. An *angle* is the union of two noncollinear rays with a common endpoint. The common endpoint is called the *vertex* of the angle, and the rays are called the *sides* of the angle.



The Ruler Postulate and Ruler Placement Postulate were motivated by the "real-world" use of rulers. A similar set of postulates, SMSG Postulates 11 −13, which are motivated by the "real-world" protractor, do the same for angles. Hence, these axioms are sometimes referred to as the *protractor postulates*.

Postulate 11. (*Angle Measurement Postulate*) To every angle there corresponds a real number between 0 and 180.

Postulate 12. (*Angle Construction Postulate*) Let $\overrightarrow{AB}$ be a ray on the edge of the half-plane *H*. For every *r* between 0 and 180, there is exactly one ray $\overrightarrow{AP}$ with *P* in *H* such that $m(\angle PAB) = r$.

Postulate 13. (*Angle Addition Postulate*) If *D* is a point in the interior of $\angle BAC$, then $m(\angle BAC) = m(\angle BAD) + m(\angle DAC)$.

Note that an angle has measure between 0 and 180. No angle has measure greater than or equal to 180, or less than or equal to 0.

Definitions.

Two angles are *congruent* if they have the same measure, denoted $\angle ABC \cong \angle DEF$.

The *interior of an angle* $\angle ABC$ is the intersection of set of all points on the same side of line *BC* as *A* and the set of all points on the same side of line *AB* as *C*, denoted $\operatorname{int}(\angle ABC)$. *(Note that this definition uses the Plane Separation Postulate.)*

The *interior of a triangle ABC* is the intersection of the set of points on the same side of line *BC* as *A*, on the same side of line *AC* as *B*, and on the same side of line *AB* as *C*.

The *bisector of an angle* $\angle ABC$ is a ray *BD* where *D* is in the interior of $\angle ABC$ and $\angle ABD \cong \angle DBC$.

A *right angle* is an angle that measures exactly 90.

An *acute angle* is an angle that measures between 0 and 90.

An *obtuse angle* is an angle that measures between 90 and 180.

Two lines are *perpendicular* if they contain a right angle.

The next theorem, stated here without proof, will be used in later sections.

Theorem 2.7. $m(\angle ABD) < m(\angle ABC)$ ***and D is on the same side of line*** $\overleftrightarrow{AB}$ ***as C if and only if*** $D \in \operatorname{int}(\angle ABC)$.

Exercise 2.32. Find the axioms from a high school book that correspond to SMSG Postulates 11, 12, and 13.

Exercise 2.33. Find the measures of the three angles determined by the points $A(1, 1)$, $B(1, 2)$ and $C(2, 1)$ where the points are in the (a) Euclidean Plane; and (b) Poincaré Half-plane. Also, find the sum of the measures of the angles of the triangles.

Exercise 2.34. Find the angle bisector of $\angle ABC$, if $A(0, 5)$, $B(0, 3)$, and $C\left(2, \sqrt{21}\right)$ where the points are in the (a) Euclidean Plane; and (b) Poincaré Half-plane.

Exercise 2.35. Given $B \in \operatorname{int}(\angle AOC)$, $C \in \operatorname{int}(\angle BOD)$, and $m(\angle AOB) = m(\angle COD)$. Prove or disprove $m(\angle AOC) = m(\angle BOD)$.

Exercise 2.36. Prove or disprove that all right angles are congruent.

Exercise 2.37. Prove or disprove that an angle has a unique bisector.

Exercise 2.38. (a) Prove that given a line and a point on the line, there is a line perpendicular to the given line and point on the line.
(b) Prove the existence of two lines perpendicular to each other.

Exercise 2.39. Prove Theorem 2.7.

Exercise 2.40. Prove congruence of angles is an equivalence relation on the set of all angles.

2.4.1 Plane Separation Postulate 2.5.1 Supplement Postulate

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