2.5.2 SAS Postulate

Euclid taught me that without assumptions there is no proof. Therefore, in any argument, examine the assumptions.
—Eric Temple Bell (1883–1960)

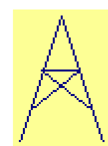
Postulate 15. (*SAS Postulate*) Given a one-to-one correspondence between two triangles (or between a triangle and itself). If two sides and the included angle of the first triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence.

We restate the Crossbar Theorem here since it plays an important role in the proofs of some of the results in this section.

Theorem 2.9. (Crossbar Theorem) If $P \in \text{int}(\angle ABC)$***, then ray BP and segment AC intersect in a unique point F and A-F-C.***

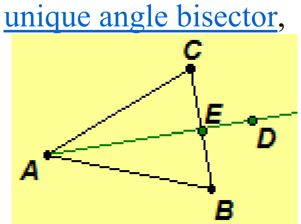
Definition.
An *isosceles triangle* is a triangle with two congruent sides. If the isosceles triangle has exactly two congruent sides, the angles opposite the two congruent sides are called *base angles,* the angle formed by the two congruent sides is called the *vertex angle*, and the third noncongruent side is called the *base*.
An *equilateral triangle* is a triangle with all sides congruent.
A *scalene triangle has no congruent sides.*

Theorem 2.10. (Pons Asinorum) The base angles of an isosceles triangle are congruent.



Pons Asinorum (Bridge of Asses) is Proposition 5 from Book 1 of Euclid's *Elements*. The name comes from the diagram, which looks like a bridge, used in Euclid's method for proving the theorem. (See the figure on the right or Byrne's Edition of Euclid's Elements.) The method used here is similar to the method used in many high school courses with one major exception. Since a proof should not be based on a picture and preconceived ideas, we need the Crossbar Theorem to fill in a gap that is not addressed in most high school courses.

Proof. Let ΔABC be an isosceles triangle with $\overline{AB} \cong \overline{AC}$. Since every angle has a unique angle bisector, let ray *AD* be the bisector of $\angle BAC$. By the Crossbar Theorem, ray *AD* and segment *BC* intersect at a unique point *E* and *B-E-C*. Thus $\angle BAE = \angle BAD \cong \angle CAD = \angle CAE$. Since congruence of segments is an equivalence relation, $\overline{AE} \cong \overline{AE}$. Hence by SAS Postulate, $\Delta ABE \cong \Delta ACE$. Thus $\angle CBA = \angle EBA \cong \angle ECA = \angle BCA$. Therefore, the base angles of an isosceles triangle are congruent.//



Exercise 2.48. Find the axiom from a high school book that corresponds to the SAS Postulate.

Exercise 2.49. Show the SAS Postulate is not satisfied by the (a) Taxicab plane; and (b) Max-distance plane. Thus showing independence.

Exercise 2.50. Prove or disprove. If quadrilateral *ABCD* is such that $\overline{BD}$ and $\overline{AC}$ intersect at a point *M* and *M* is the midpoint of both $\overline{BD}$ and $\overline{AC}$, then $\overline{AB}$ is congruent to $\overline{CD}$.

Exercise 2.51. Prove or disprove. If quadrilateral *ABCD* has $\overline{CD}$ congruent to $\overline{CB}$ and ray *CA* is the bisector of $\angle DCB$, then $\overline{AB}$ is congruent to $\overline{AD}$.

Exercise 2.52. Prove or disprove. If $\square ABCD$ is a quadrilateral and $\overline{AB} \cong \overline{AD}$, then $\angle D \cong \angle B$.

Exercise 2.53. State and prove each theorem. (a) SSS Theorem; and (b) ASA Theorem.

Exercise 2.54. Prove that all points equidistant from two points *A* and *B* are on the perpendicular bisector of $\overline{AB}$.