Euclidean Parallel Postulate

This ought even to be struck out of the Postulates altogether; for it is a theorem involving many difficulties..
—Proclus (410–485)

Euclid's Fifth Postulate. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

SMSG Postulate 16. (*Euclidean Parallel Postulate*) Through a given external point there is at most one line parallel to a given line.

Playfair's Axiom. Through a point not on a line there is exactly one line parallel to the given line.

Euclidean Proposition 2.1. There exists a line and a point not on the line such that there is a unique line through the point that is parallel to the line.

Euclidean Proposition 2.2. If A and D are points on the same side of line BC and line BA is parallel to line CD, then $m(\angle ABC) + m(\angle BCD) = 180$.

Euclidean Proposition 2.3. If l_1, l_2, l_3 are three distinct lines such that l_1 is parallel to l_2 and l_2 is parallel to l_3, then l_1 is parallel to l_3.

Euclidean Proposition 2.4. If l_1, l_2, l_3 are three distinct lines such that l_1 intersects l_2 and l_2 is parallel to l_3, then l_1 intersects l_3.

Euclidean Proposition 2.5. A line perpendicular to one of two parallel lines is perpendicular to the other.

Euclidean Proposition 2.6. If l_1, l_2, l_3, l_4 are four distinct lines such that l_1 is parallel to l_2, l_3 is perpendicular to to l_1, and l_4 is perpendicular to l_2, then l_3 is parallel to l_4.

Euclidean Proposition 2.7. Every two parallel lines have a common perpendicular.

Euclidean Proposition 2.8. The perpendicular bisectors of the sides of a triangle intersect at a point.

Euclidean Proposition 2.9. There exists a circle passing through any three noncollinear points.

Euclidean Proposition 2.10. There exists a point equidistant from any three noncollinear points.

Euclidean Proposition 2.11. A line intersecting and perpendicular to one side of an acute angle intersects the other side.

Euclidean Proposition 2.12. Through any point in the interior of an angle there exists a line intersecting both sides of the angle not at the vertex.

Euclidean Proposition 2.13. If two parallel lines are cut by a transversal, then the alternate interior angles are congruent. *(The converse of Theorem 2.15.)*

Euclidean Proposition 2.14. The sum of the measures of the angles of any triangle is 180.

Euclidean Proposition 2.15. There exists a triangle such that the sum of the measures of the angles of the triangle is 180.

Euclidean Proposition 2.16. The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles. *(Compare to the Exterior Angle Theorem.)*

Euclidean Proposition 2.17. If a point C is not on $\overline{AB}$ but on the circle with diameter $\overline{AB}$, then $\angle ACB$ is a right angle.

Euclidean Proposition 2.18. If $\angle ACB$ is a right angle, then C is on the circle with diameter $\overline{AB}$.

Euclidean Proposition 2.19. The perpendicular bisectors of the legs of a right triangle intersect.

Euclidean Proposition 2.20. There exists an acute angle such that every line intersecting and perpendicular to one side of the angle intersects the other side.

Euclidean Proposition 2.21. There exists an acute angle such that every point in the interior of the angle is on a line intersecting both sides of the angle not at the vertex.

Euclidean Proposition 2.22. If l_1, l_2, l_3, l_4 are four distinct lines such that l_1 is perpendicular to l_2, l_2 is perpendicular to l_3, and l_3 is perpendicular to l_4, then l_1 intersects l_4.

Euclidean Proposition 2.23. There exists a rectangle.

Euclidean Proposition 2.24. There exist two lines equidistant from each other.

Euclidean Proposition 2.25. If three angles of a quadrilateral are right angles, then so is the fourth.

Euclidean Proposition 2.26. There exists a pair of similar triangles that are not congruent.

Euclidean Proposition 2.27. The diagonals of a Saccheri quadrilateral bisect each other.

Euclidean Proposition 2.28. One of the upper base angles of a Saccheri quadrilateral is a right angle.

Euclidean Proposition 2.29. Any three lines have a common transversal.

Euclidean Proposition 2.30. There do not exist three lines such that each two are on the same side of the third.

Euclidean Proposition 2.31. In $\triangle ABC$, if M is the midpoint of segment AB and N is the midpoint of segment AC, then the length of segment MN is equal to half the length of segment BC.