

## 4.6.1 Definitions for Perspectivity and Projectivity

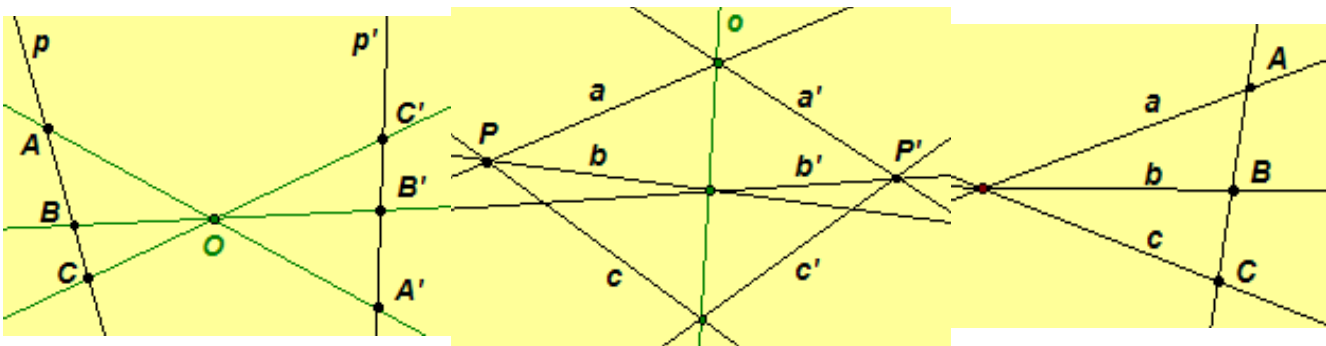
*Mighty is geometry; joined with art, resistless.*

—  [Euripides \(480–406 B.C.\)](#)

- A one-to-one mapping between a pencil of points and a pencil of lines is called an *elementary correspondence* if each point of the pencil of points is incident with the corresponding line of the pencil of lines. The elementary correspondence is denoted  $X \bar{\wedge} x$  or  $x \bar{\wedge} X$ . [An elementary correspondence is also called a *perspectivity* between a pencil of points and a pencil of lines.]
- A one-to-one mapping between two pencils of points is called a *perspectivity* if the lines incident with the corresponding points of the two pencils are concurrent. The point where the lines intersect is called the *center of the perspectivity*. The perspectivity is denoted  $X \overset{O}{\wedge} X'$  where  $O$  is the center of perspectivity.
- A one-to-one mapping between two pencils of lines is called a *perspectivity* if the points of intersection of the corresponding lines of the two pencils are collinear. The line containing the points of intersection is called the *axis of the perspectivity*. The perspectivity is denoted  $x \overset{o}{\wedge} x'$  where  $o$  is the axis of perspectivity.

Note that a perspectivity is a composition of two elementary correspondences between either two pencils of points or two pencils of lines.

Illustrations of perspectivities:  $ABC \overset{O}{\wedge} A'B'C'$ ,  $abc \overset{o}{\wedge} a'b'c'$ , and an elementary correspondence  $ABC \bar{\wedge} abc$ .

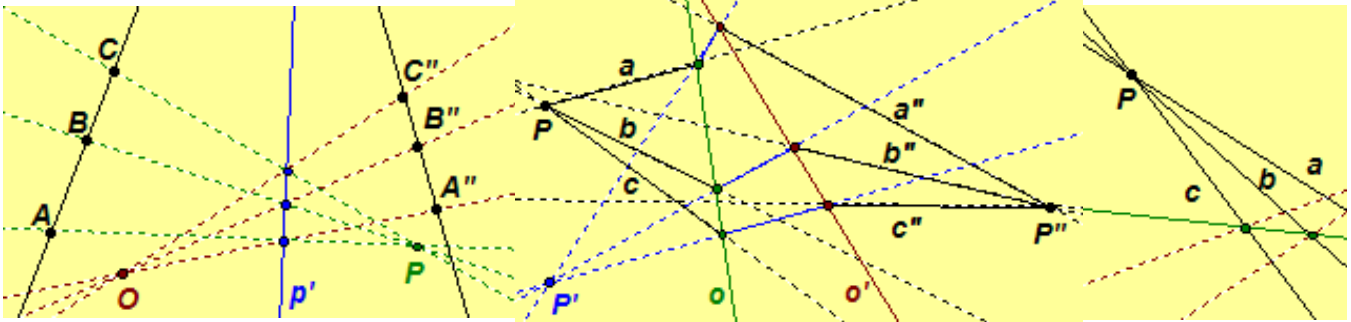


Click here for a dynamic illustrations of perspectivity [GeoGebra](#) or [JavaSketchpad](#).

- A one-to-one mapping between two pencils of points is called a *projectivity* if the mapping is a composition of finitely many elementary correspondences or perspectivities. A projectivity is denoted  $X \wedge X'$  or  $x \wedge x'$  or  $x \wedge X$ .

When a projectivity exists between two pencils, the pencils are said to be *projectively related*. Also, note that elementary correspondences and perspectivities themselves are projectivities.

Illustrations of projectivities: Figure 1  $ABC \wedge A''B''C''$ , Figure 2  $abc \wedge a''b''c''$ , and Figure 3  $abc \wedge A''B''C''$ .



Click here to explore dynamic illustrations of projectivity [GeoGebra](#) or [JavaSketchpad](#).

To better see the projectivities in each figure, we describe a path to follow beginning with one of the points/lines and following the "path of its projection."

- In Figure 1, from  $A$  follow the path with center  $P$  to axis  $p'$ , from axis  $p'$  follow the corresponding path with center  $O$  to  $A''$ .
- In Figure 2, follow the path  $a$  from center  $P$  to axis  $o$ , from axis  $o$  follow the corresponding path from center  $P'$  to axis  $o'$  which leads to the corresponding path  $a''$  with center  $P''$ .
- In Figure 3, follow the path  $a$  from center  $P$  to axis  $o$ , then from axis  $o$  follow the corresponding path through center  $Q$  to  $A''$ .

**Exercise 4.28.** Symbolize each perspectivity forming the projectivity in each of the above diagrams.

**Exercise 4.29.** Find the image of the point  $D$  or line  $d$  for each projectivity.

