4.1.2 Historical Overview

Creative mathematicians now, as in the past, are inspired by the art of mathematics rather than by any prospect of ultimate usefulness.
—Eric Temple Bell (1883–1960)

Projective geometry began during the Renaissance period, including the 14th century through the 16th century, when Renaissance artists were attempting new techniques to achieve a sense of depth and three-dimensional space on a two-dimensional surface. Before this time, paintings and drawings rarely displayed a sense of depth due to how parallel lines were displayed on canvases. Artists would display parallel lines, such as curbs on a street, as parallel in their work causing a distorted depiction of what a person actually sees.

The attempt to achieve a realistic portrayal of the world on a two-dimensional surface was studied by many artists during the Renaissance period. One such artist was Albrecht Dürer (1471–1528). Dürer was a well-known German artist who worked in such areas as painting, wood carving, and engraving. He strived towards an exact display of the world around him; this goal drove Dürer toward the study of geometry. He worked on discovering geometric rules for depicting three-dimensional objects on a plane that would give the viewer a sense of depth.

Girard Desargues (1591–1661), a French geometer, engineer, and army officer, was the first person to prove properties relating to projective geometry. He wrote a treatise and theorems that helped create projective geometry and led to Desargues Theorem, a theorem named in his honor. His work was ignored and ridiculed while he was alive; it was not until almost two centuries later, before another French geometer, Michel Chasles (1793–1880), discovered a copy of Desargues' work and re-released the work and ideas to the mathematics community.

There are two main reasons why historians of mathematics believe Desargues' work in this new branch of geometry was ignored and left to be lost for almost two hundred years. One suggested reason is the new geometry that René Descartes (1596–1650) had developed just two years earlier. As mentioned in the previous chapter, Descartes was a pioneer in the development of analytical geometry. Most mathematicians, during this time, worked on the development of new ideas and theorems in analytical geometry. The other suggested reason for the initial disregard towards Desargues work in projective geometry was his unconventional writing style. In his treatise, Desargues introduced over sixty new terms, many involving natural items, such as flowers and trees, as names for specific geometric objects. This technique made the paper extremely difficult to read and understand. He was ridiculed by many mathematicians who read his work. The one term that has survived from his original work is *involution*, which is thought to have survived because it was the word that received the most criticism from others who critiqued his work. *(An involution is a bijection of period two. In projective geometry, an involution is a projectivity of period two.)*

Blaise Pascal (1623–1662), a French geometer, physicist, inventor, and philosopher, was a contemporary of Desargues. Pascal was educated at home by his father, and by the age of seventeen, had already done a significant amount of work in mathematics, including composing a paper on conic sections. Pascal was well known by prestigious mathematicians such as René Descartes. Pascal worked from Desargues' prior work, to help clarify Desargues' ideas on conic sections; however, Pascal's paper is now lost. Pascal "has been described as the greatest 'might-have-been' in the history of

mathematics" (Eves 243). He discontinued the study of mathematics, while suffering from poor health, to devote his life to religion. He returned to the study of mathematics only for a brief period while suffering from a toothache. While in pain from his tooth, Pascal thought of a geometric idea and his tooth stopped aching. Believing this to be a sign from God, Pascal briefly returned to the study of mathematics. Later in his life, Pascal is credited with inventing the first mechanical device that would add and subtract by the use of wheels for calculations.

Conics are closely related to results in projective geometry. The study of conics first arose from the attempts to solve the problem of duplicating the cube, which is one of three famous problems of antiquity (trisecting the angle, duplicating the cube, and squaring the circle). The work on duplication of a cube led Menaechmus (375–325 B.C.) to discover the conic sections. Menaechmus, a Greek geometer, is credited with discovering conics in his solution to the problem of doubling the cube.

Apollonius of Perga (262–190 B.C.), also a Greek geometer, is one of the original students of mathematical astronomy and worked on a range of other mathematical subjects. Although Apollonius contributed to a range of mathematical areas, his greatest and most noted achievement was his work on conics, an eight volume book titled *Conic Sections*. *Conic Sections* contains approximately 400 propositions that worked from and went beyond the results of his predecessors, Menaechmus and Euclid. His work helped future mathematicians such as Desargues and Pascal. He is also credited with giving the names ellipse, parabola, and hyperbola. After Apollonius, the study of conics declined.

Johannes Kepler (1571–1630), a German astronomer and mathematician, renewed interest in conics with his work on planetary motion. He showed how the parabola can be considered the limiting case of both the ellipse and hyperbola, where one focus is moved to infinity.

Jean-Victor Poncelet (1788–1867), a French mathematician, is considered to be one of the fathers of modern geometry (the other being Joseph Diez Gergonne) and has had a significant impact in the field of projective geometry. Along with being a mathematician, Poncelet served in the French army and was taken prisoner while fighting in Napoleon's Russian campaign. During his two year captivity, he worked on different mathematics problems, some within the area of projective geometry, building off the ideas of Desargues and Pascal. The treatise on which he worked while imprisoned was not published for some 50 years. His paper gave enormous momentum towards the study of projective geometry.

Joseph Diez Gergonne (1771–1859), a French artillery officer and professor of mathematics, introduced the term *polar* in its modern projective geometry sense in 1813. The principle of duality grew out of his and Poncelet's work and was first stated by Gergonne in 1826.

Jakob Steiner (1796–1863), a Swiss geometer, also had a significant impact on projective geometry and built on Poncelet's work. Steiner did not learn to write until he was fourteen and was unschooled until he was eighteen. Steiner discovered the Steiner surface and the Steiner theorem. His continuing work on Poncelet's work also developed into the Poncelet-Steiner theorem (if given a circle and its center, whatever can be constructed by straightedge and compass together can be constructed by straightedge alone). These ideas and theorems encouraged the growth of projective geometry, including relationships with conics and projective geometry.

Although much of the momentum to study projective geometry and the ideas behind it originally came from artists' desire for depth and realism in their art work, it has now grown into a huge branch of geometry. Today, projective geometry is used in various trades including computer design, statistical design theory, photography, and, of course, what started it all, art.